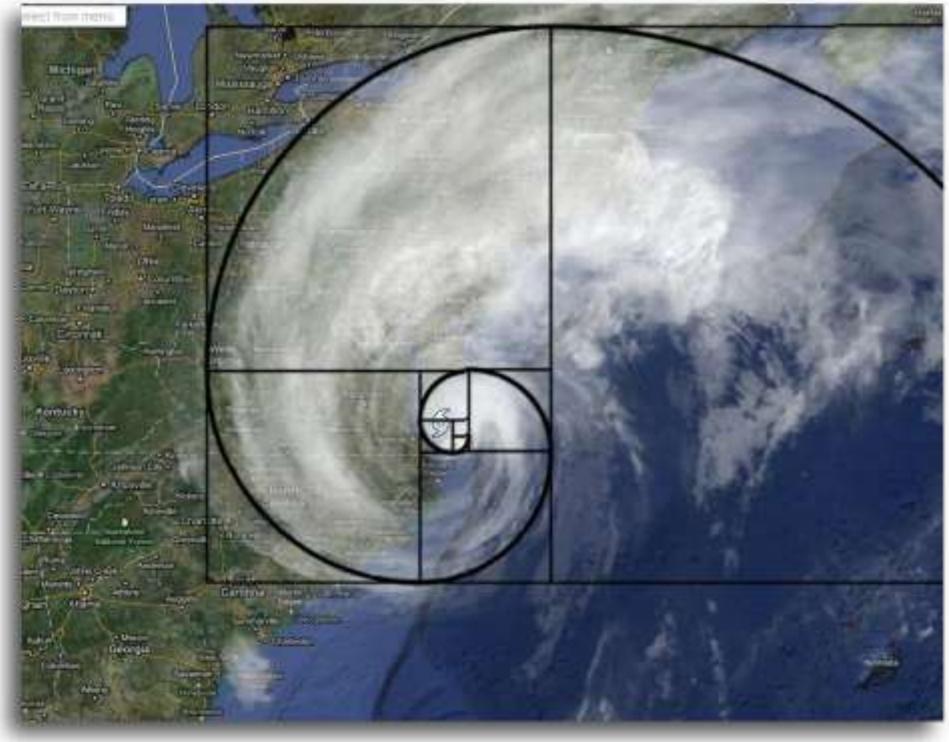
Esponenziali e Logaritmi



La funzione esponenziale

$a^b=c$

$x^b = y$ $a^x = y$

- $a=0$ $0^x = 0$ (indefinita per $x=0$)
- $a=1$ $1^x = 1$
- $a<0$ a^x (cambia segno di continuo)

La funzione esponenziale

$y=a^x$ con $a<0$

$$(-1)^{\frac{6}{4}} = (-1)^{\frac{3}{2}}$$

$$\sqrt[4]{(-1)^6} = \sqrt[2]{(-1)^3}$$

La funzione esponenziale

$y=a^x$

$a>0$ e $a\neq 1$

$0 < a < 1$

$1 < a < +\infty$

$\forall x \in R$

L'equazione esponenziale

$a^x = q$

$\begin{cases} y = a^x \\ y = q \end{cases}$

$q \leq 0$ Nessuna soluzione

$q > 0$ 1 soluzione

$a^x > 0, \quad \forall x \in R$

L'equazione esponenziale

$2^x=4$	$x=2$
$5^x=125$	$x=3$
$2^x=\frac{1}{8}$	$x=-3$
$\left(\frac{3}{4}\right)^x=1$	$x=0$
$\left(\frac{7}{8}\right)^x=5$	$x=?$

Il logaritmo

$a^x = y$ $\qquad$ $x = \log_a y$

base: 10, e $\qquad$ argomento

$a > 0$ e $a \neq 1$

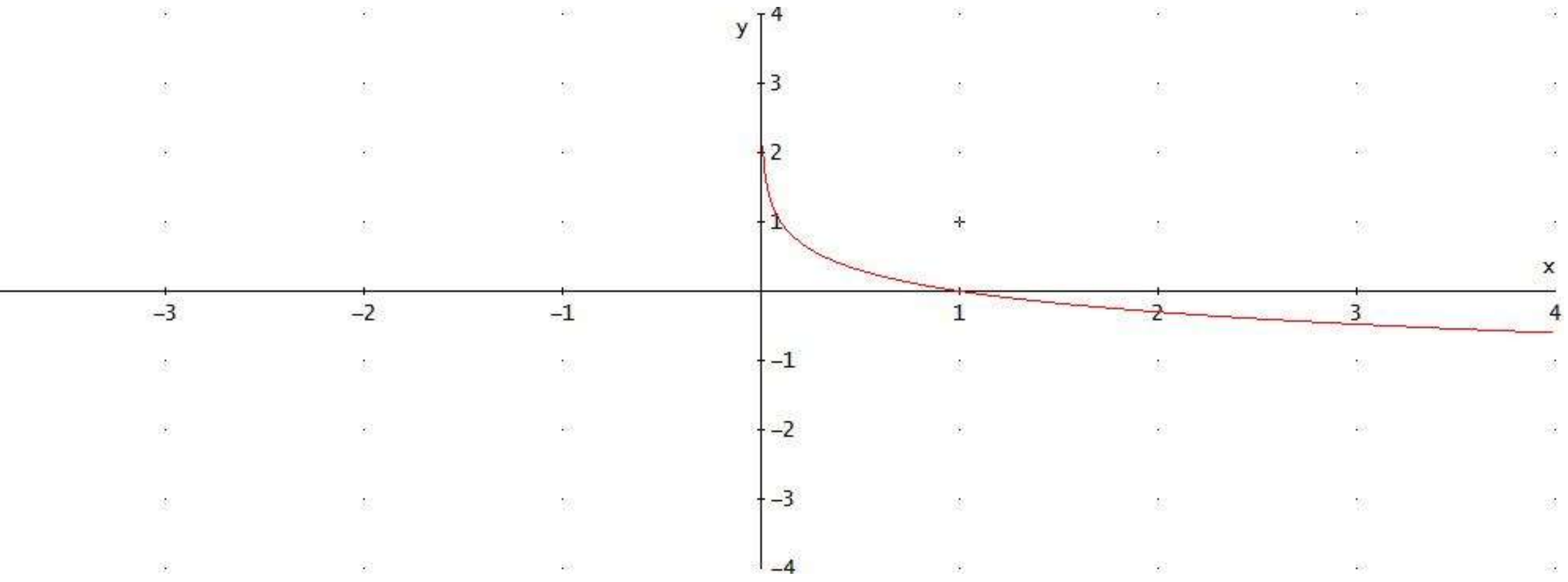
$y > 0$

$\left(\frac{7}{8}\right)^x = 5$ $\qquad$ $x = \log_{\frac{7}{8}} 5$

La funzione logaritmo

$y = \log_a x \qquad 0 < a < 1$

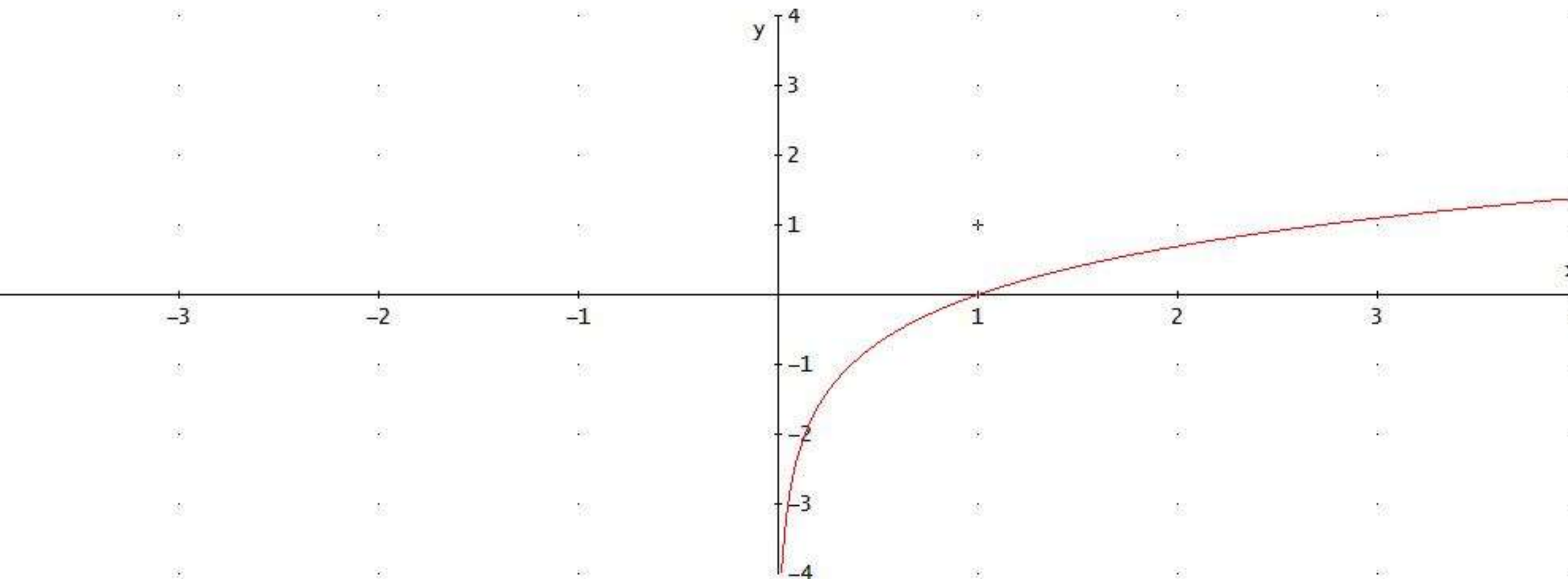
$y = \log_{1/10} x \qquad x > 0$



La funzione logaritmo

$y = \log_a x \qquad 1 < a < +\infty$

$y = \mathrm{Log}\, x \qquad x > 0$

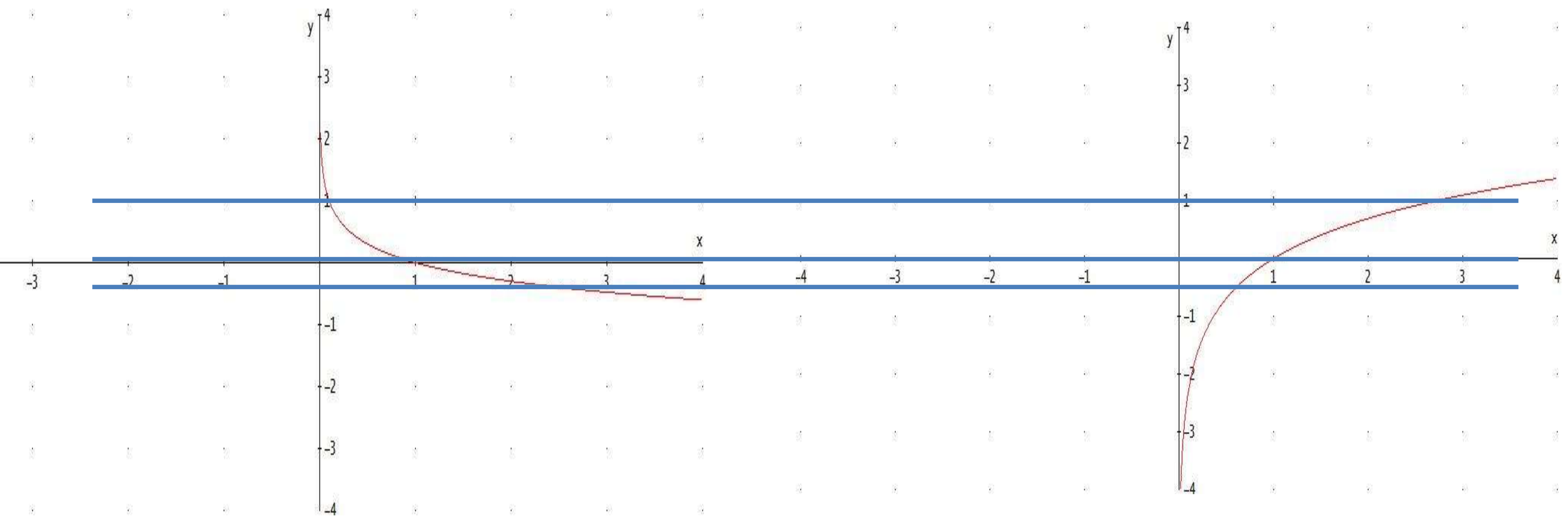


L'equazione logaritmica

$\log_a x = q$

$$\begin{cases} y = \log_a x \\ y = q \end{cases}$$

$\forall q \in R$ 1 soluzione



L'equazione logaritmica

$y = \log_3 9$	y=2
$y = \log_3 \frac{1}{9}$	y=-2
$y = \log_2 1$	y=0
$y = \log_2 2$	y=1

Proprietà del logaritmo

$\log_a a = 1$	I N	$a^1 = a$
$\log_a 1 = 0$	F A	$a^0 = 1$
$\log_a a^x = x$	T T	$a^x = a^x$
$a^{\log_a x} = x$	I	$\log_a x$ è l'esponente da dare ad a per ottenere x

Proprietà del logaritmo

$$\log_a X + \log_a Y = \log_a X \cdot Y$$

$$\log_a X = x \qquad \Rightarrow \qquad X = a^x$$

$$\log_a Y = y \qquad \qquad Y = a^y$$

$$X \cdot Y = a^x \cdot a^y = a^{x+y}$$

$$\log_a X \cdot Y = \log_a a^{x+y} = x + y = \log_a X + \log_a Y$$

Proprietà del logaritmo

$$\log_a X - \log_a Y = \log_a \frac{X}{Y}$$

$$\log_a X = x \qquad \Rightarrow \qquad X = a^x$$

$$\log_a Y = y \qquad\qquad Y = a^y$$

$$\frac{X}{Y} = a^x : a^y = a^{x-y}$$

$$\log_a \frac{X}{Y} = \log_a a^{x-y} = x - y = \log_a X - \log_a Y$$

Proprietà del logaritmo

$$\boxed{c\log_a X = \log_a X^c}$$

$$\log_a X = x \quad \Rightarrow \quad X = a^x$$

$$X^c = (a^x)^c = a^{xc}$$

$$\log_a X^c = \log_a a^{cx} = cx = c\log_a X$$

Proprietà del logaritmo

$$\log_a X = \frac{\log_b X}{\log_b a}$$

$$\log_a X \cdot \log_b a = \log_b X$$

$$\log_a X = x \qquad \log_b a = y$$

$$X = a^x \qquad a = b^y$$

$$X = a^x = (b^y)^x = b^{yx}$$

$$\log_b X = \log_b b^{xy} = xy = \log_a X \cdot \log_b a$$

Esempio

$$y = 3\log_2 \frac{1}{8} - \log_3 \frac{1}{9} + \ln e^3 - \log_5 15 +$$

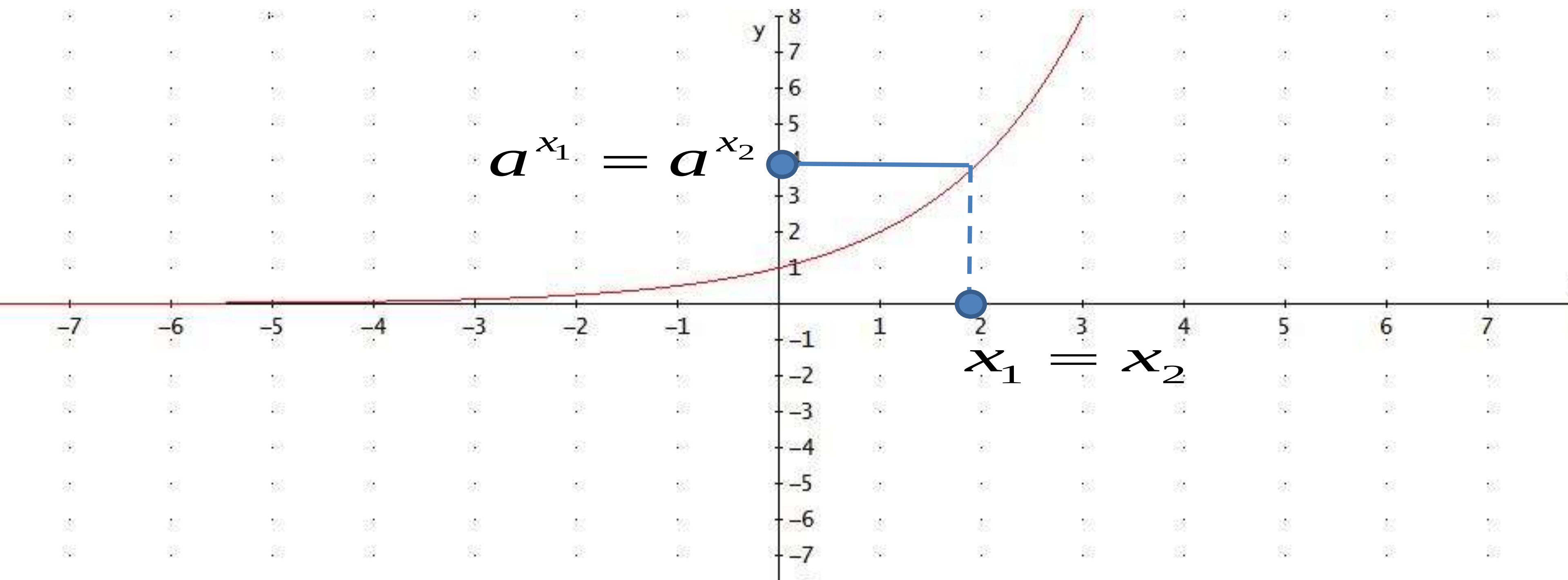
$$Log \frac{1}{1000} + \log_5 45 + \log_5 9 - 4\log_5 3 + \log_5 75$$

Equazioni esponenziali elementari

$$a^{x_1} = a^{x_2}$$

$$\Downarrow$$

$$x_1 = x_2$$



Equazioni esponenziali elementari

$$a^{f(x)} = a^{g(x)}$$

$$\Downarrow$$

$$\log_a a^{f(x)} = \log_a a^{g(x)}$$

$$\Downarrow$$

$$f(x) = g(x)$$

$$5^{2-x^2} = 5^{2x+2} \qquad 3^{2-8x} = 9^{3x+1}$$

Equazioni esponenziali elementari

$$a^{f(x)} = b^{g(x)}$$

$$\Downarrow$$

$$\log_a a^{f(x)} = \log_a b^{g(x)}$$

$$f(x) = g(x) \cdot \log_a b$$

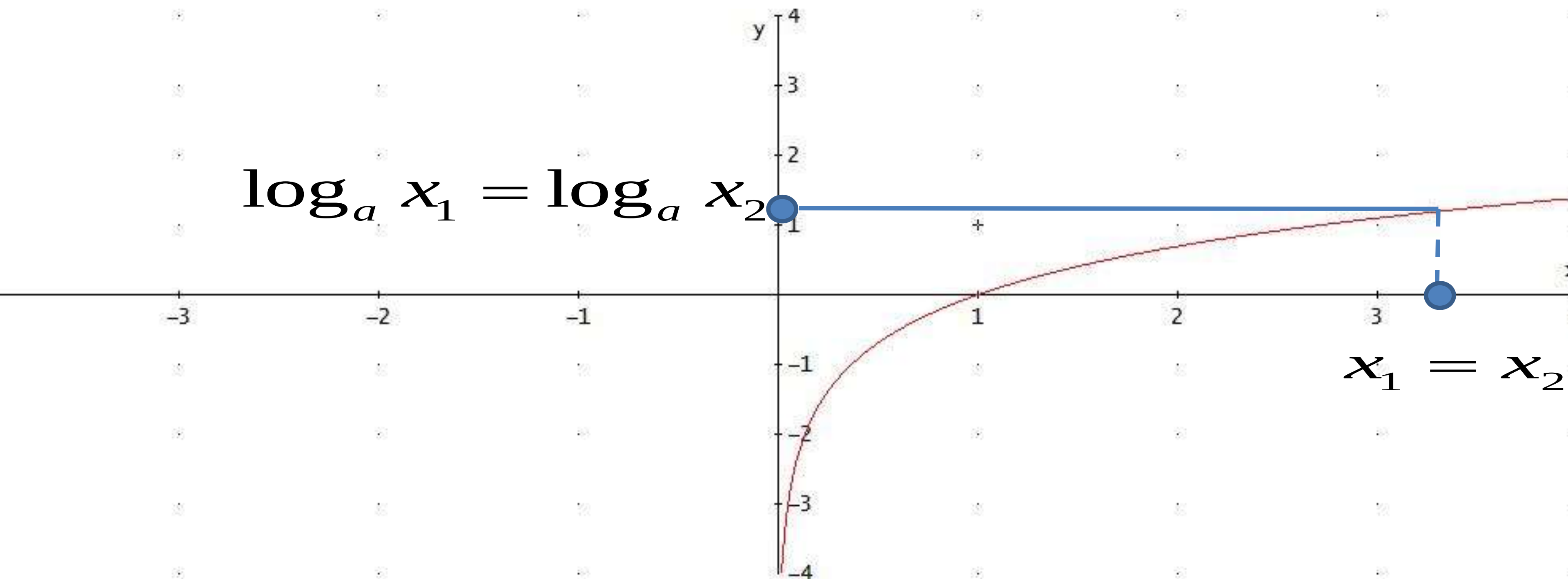
$$2^{x+1} = 5^{1-x}$$

$$2^{x-3} = 3^{x-3}$$

Equazioni logaritmiche elementari

$$\log_a x_1 = \log_a x_2$$

$$x_1 = x_2$$



Equazioni logaritmiche elementari

$$\log_a f(x) = \log_a g(x)$$

$$\Downarrow$$

$$a^{\log_a f(x)} = a^{\log_a g(x)}$$

$$\Downarrow$$

$$f(x) = g(x)$$

ATTENZIONE:

$$\begin{cases} f(x) > 0 \\ g(x) > 0 \end{cases}$$

$$\log_3(x+3) = \log_3(3x-1)$$

$$\log_3(2x+4) = 2$$

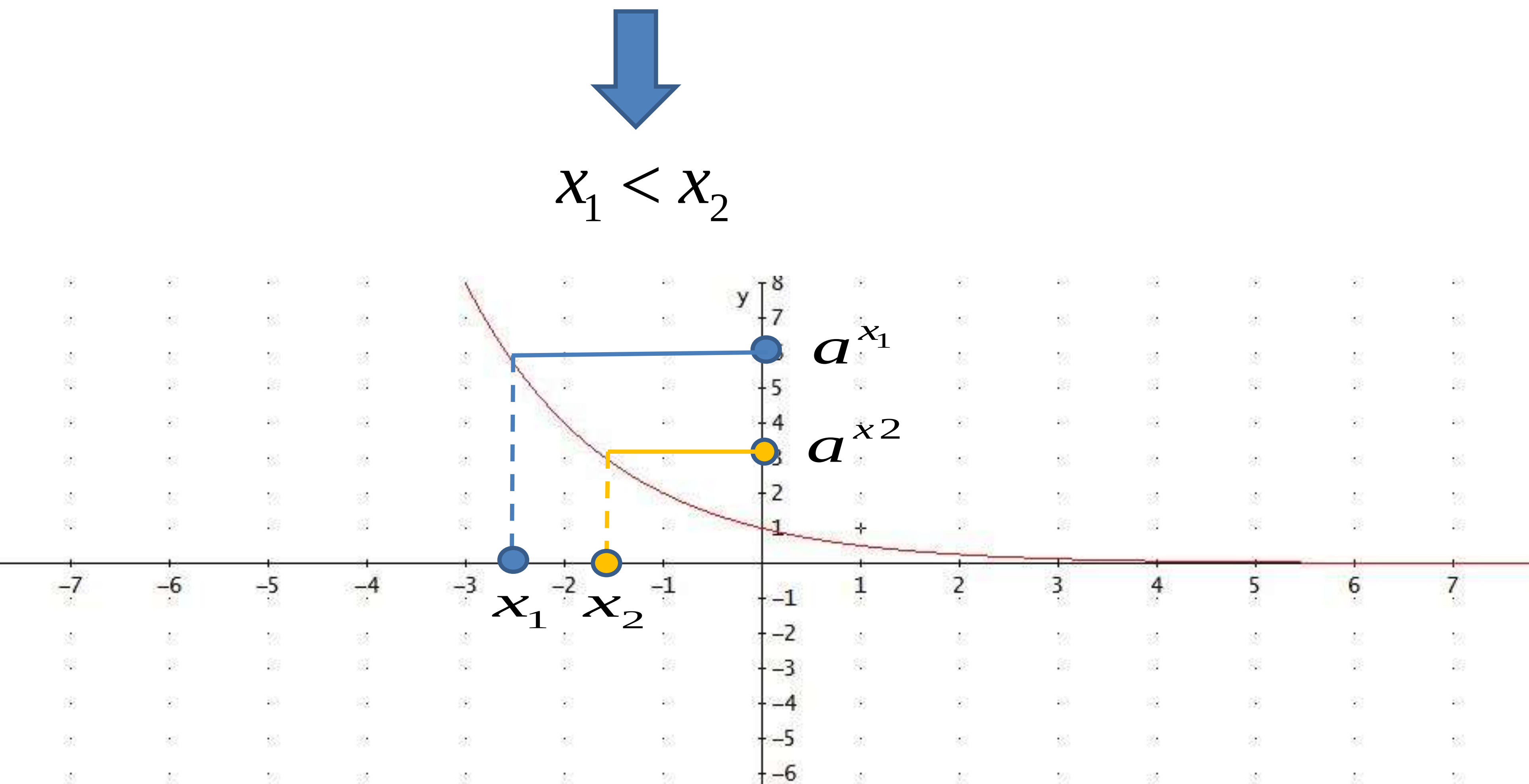
$$\log_2 x + 3\log_4 x = 10$$

Disequazioni esponenziali elementari

$$a^{x_1} > a^{x_2}$$

0<a<1

$$x_1 < x_2$$

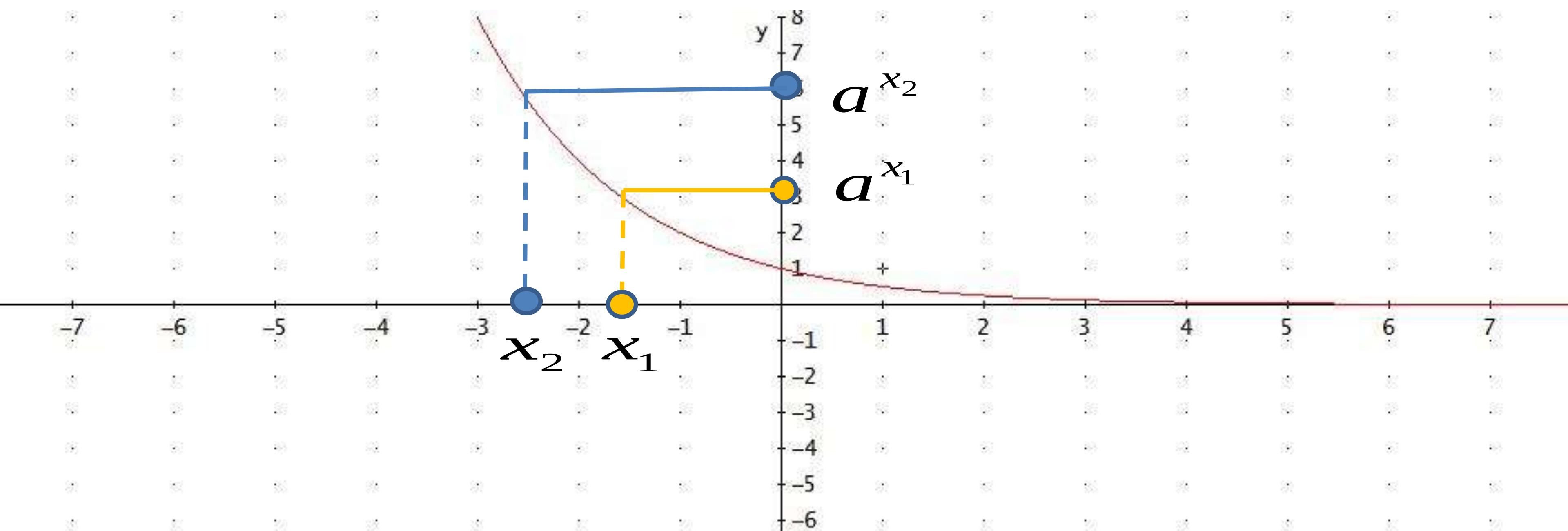


Disequazioni esponenziali elementari

$$a^{x_1} < a^{x_2}$$

$0<a<1$

$$x_1 > x_2$$

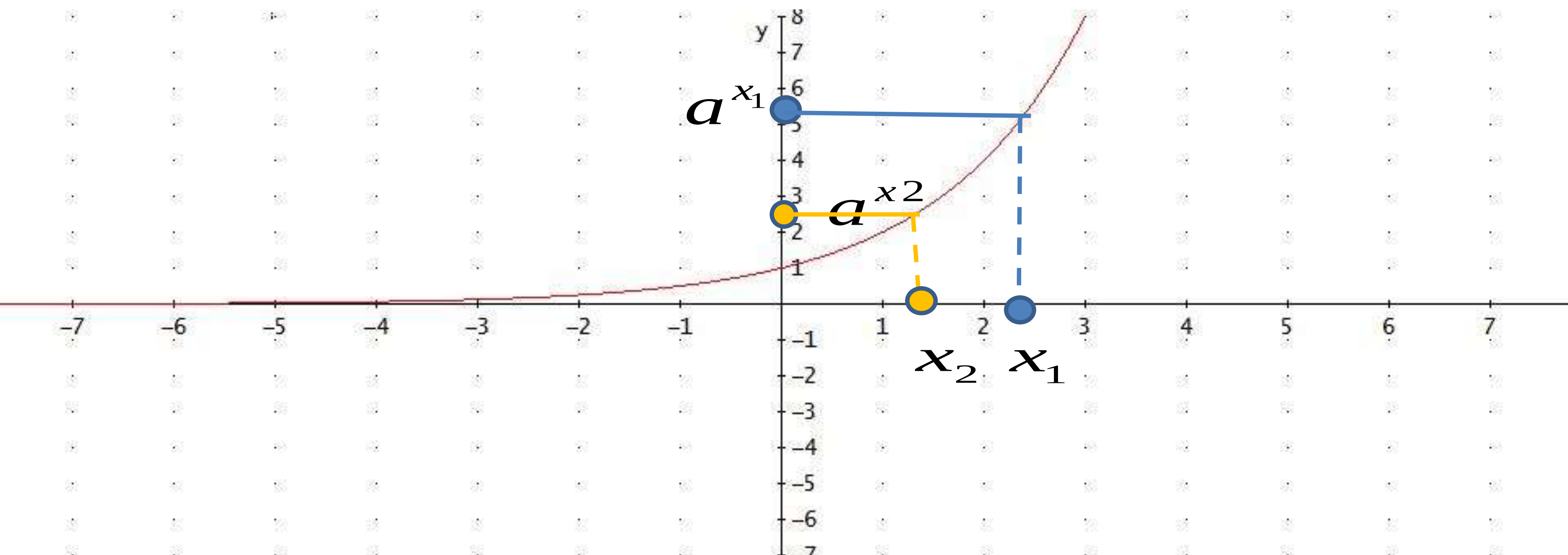


Disequazioni esponenziali elementari

$$a^{x_1} > a^{x_2}$$

a>1

$$x_1 > x_2$$

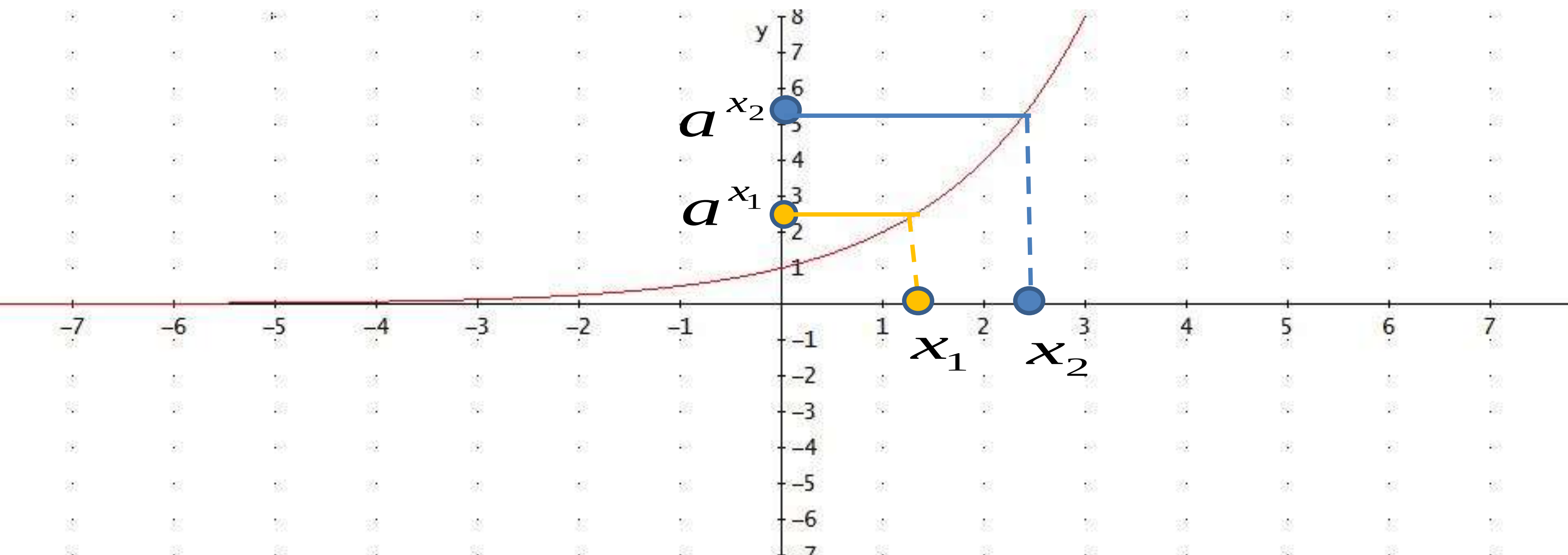


Disequazioni esponenziali elementari

$$a^{x_1} < a^{x_2}$$

a>1

$$x_1 < x_2$$



Disequazioni esponenziali

0<a<1

$$a^{f(x)} > a^{g(x)}$$

⇓

$$f(x) < g(x)$$

$$a^{f(x)} < a^{g(x)}$$

⇓

$$f(x) > g(x)$$

$$\left(\frac{3}{4}\right)^{2-x} < \left(\frac{3}{4}\right)^{3+2x}$$

Disequazioni esponenziali

a>1

$$a^{f(x)} > a^{g(x)}$$

$$\Downarrow$$

$$f(x) > g(x)$$

$$a^{f(x)} < a^{g(x)}$$

$$\Downarrow$$

$$f(x) < g(x)$$

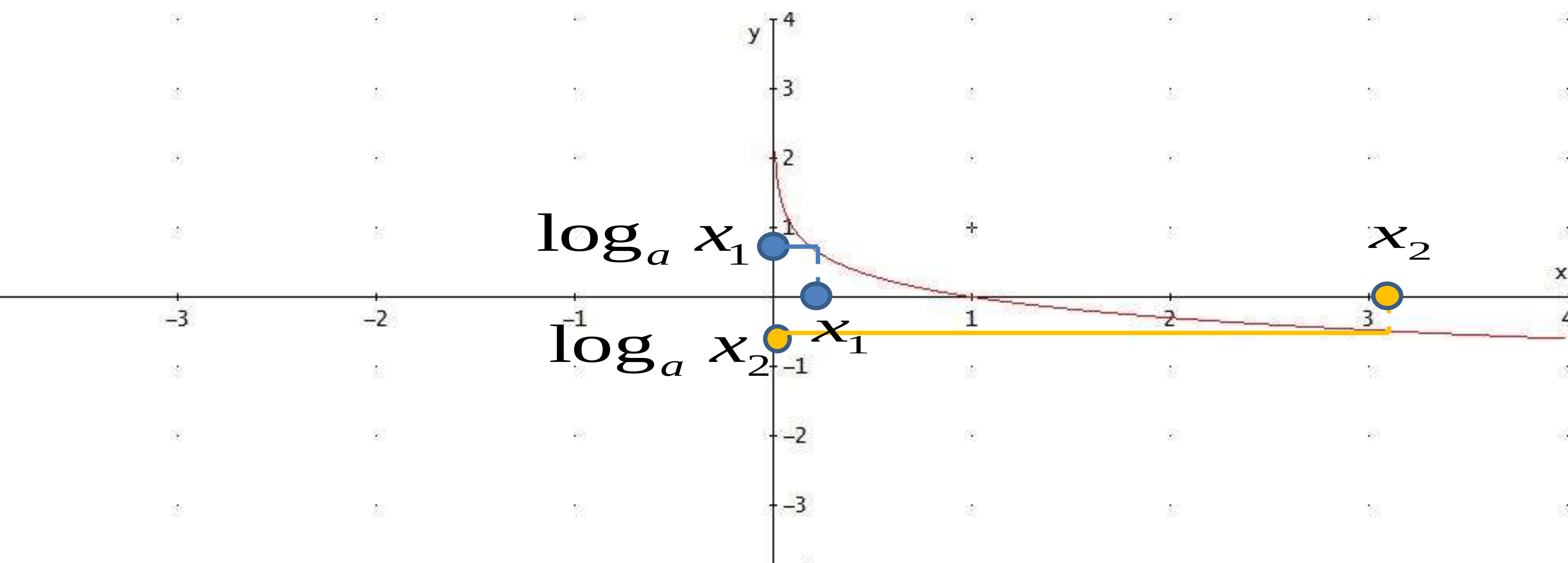
$$3^{1-4x} > 9^{x+1}$$

Disequazioni logaritmiche

$$\log_a x_1 > \log_a x_2$$

0<a<1

$$x_1 < x_2$$

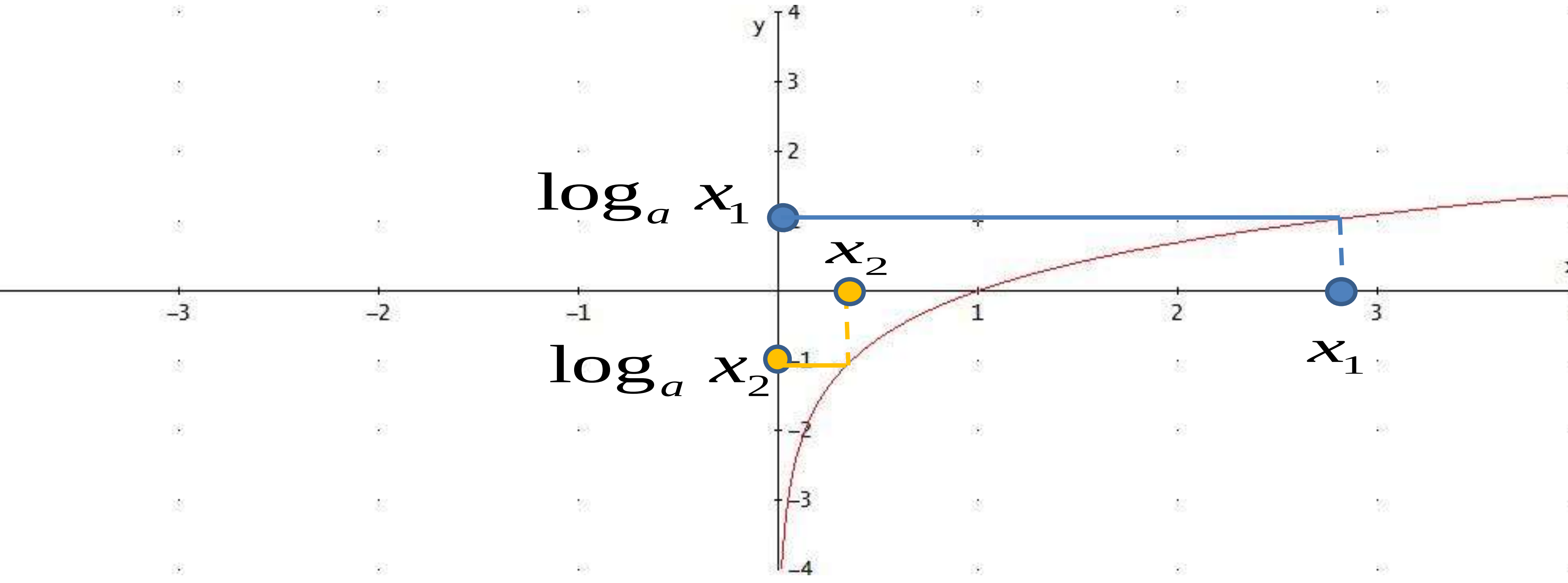


Disequazioni logaritmiche

$$\log_a x_1 > \log_a x_2$$

a>1

$$x_1 > x_2$$



Disequazioni logaritmiche

$0<a<1$

$$\log_a f(x) > \log_a g(x) \quad \Rightarrow \quad f(x) < g(x)$$

$$\log_a f(x) < \log_a g(x) \quad \Rightarrow \quad f(x) > g(x)$$

$$\log_{\frac{1}{2}}(3x+5) > 1$$

Disequazioni logaritmiche

a>1

$$\log_a f(x) > \log_a g(x) \qquad \log_a f(x) < \log_a g(x)$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$f(x) > g(x) \qquad f(x) < g(x)$$

$$\log_2(1 - x^2) < 1$$

Disequazioni tra esponenziali aventi base diversa

$$a^{f(x)} > b^{g(x)}$$

$$\Downarrow$$

$$\log_a a^{f(x)} > \log_a b^{g(x)}$$

a>1

$$\Downarrow$$

$$f(x) > g(x)\log_a b$$

$$2^{x+1} \geq 5^{1-x}$$

Disequazioni tra esponenziali aventi base diversa

$$a^{f(x)} > b^{g(x)}$$

⇓

$$\log_a a^{f(x)} < \log_a b^{g(x)}$$

⇓

$$f(x) < g(x)\log_a b$$

0<a<1

$$\left(\frac{1}{2}\right)^{2x-1} < 3^{3-x}$$