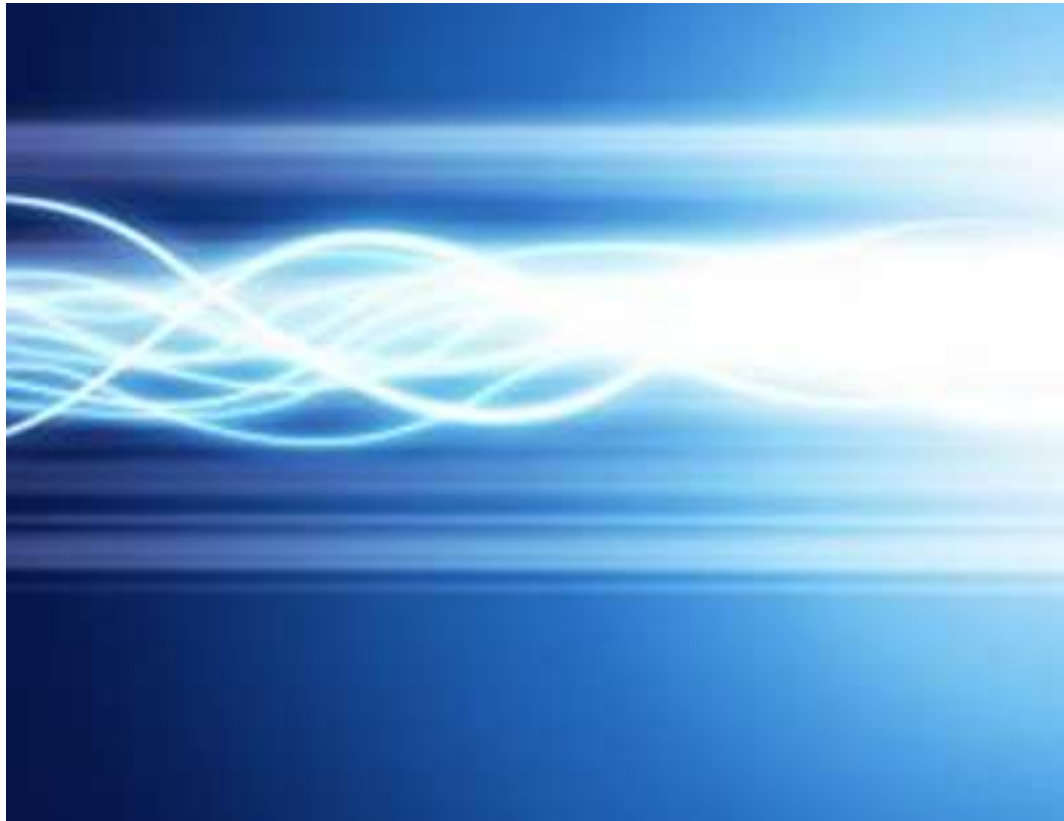
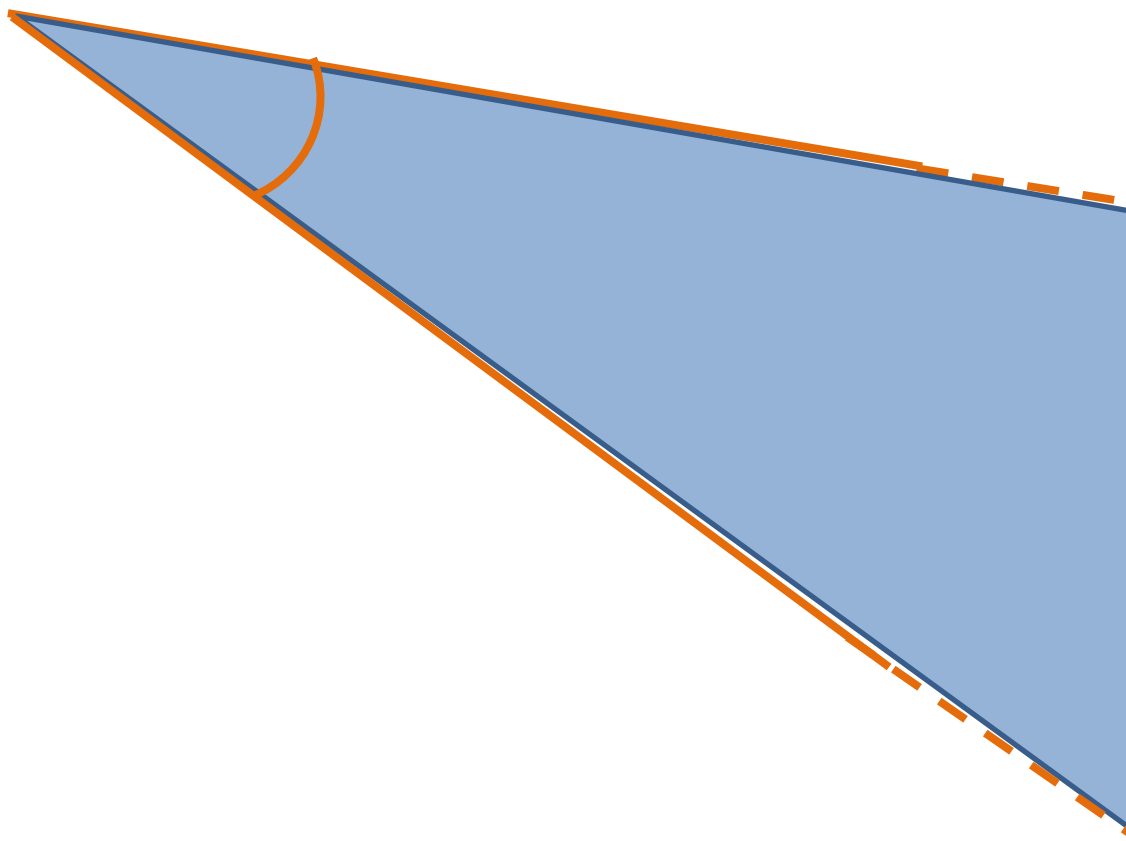


Trigonometria



La misura degli angoli



La misura degli angoli

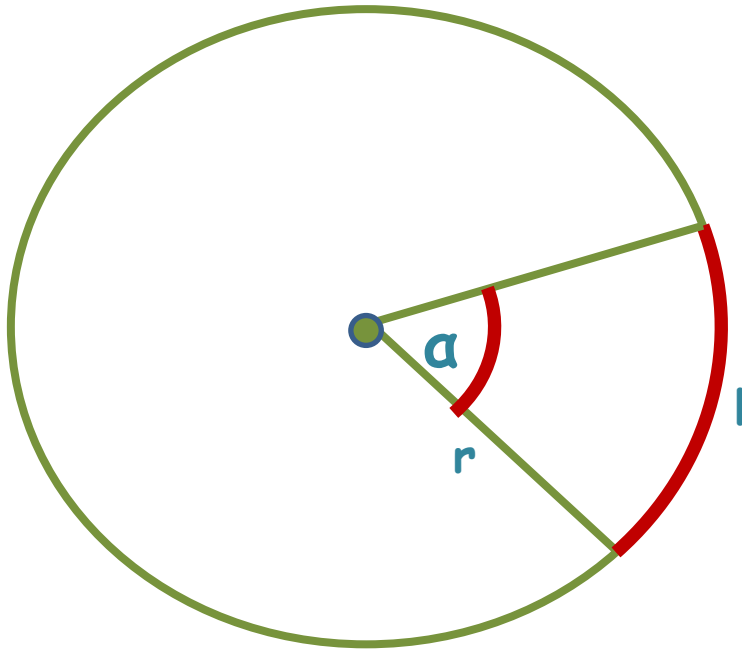
- Gradi sessagesimali
- Gradi centesimali
- Radianti

I radianti

Il radiante è quell'arco che rettificato è uguale al raggio

Un radiante è la misura di un angolo il cui arco corrispondente è lungo quanto il raggio della circonferenza cui l'arco appartiene.

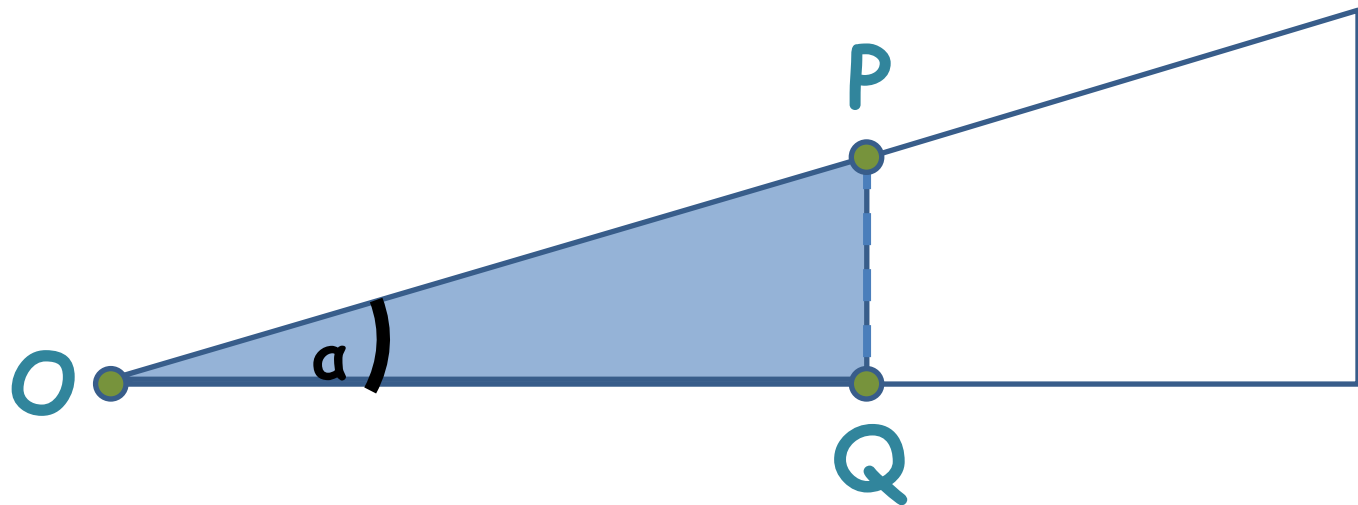
I radianti



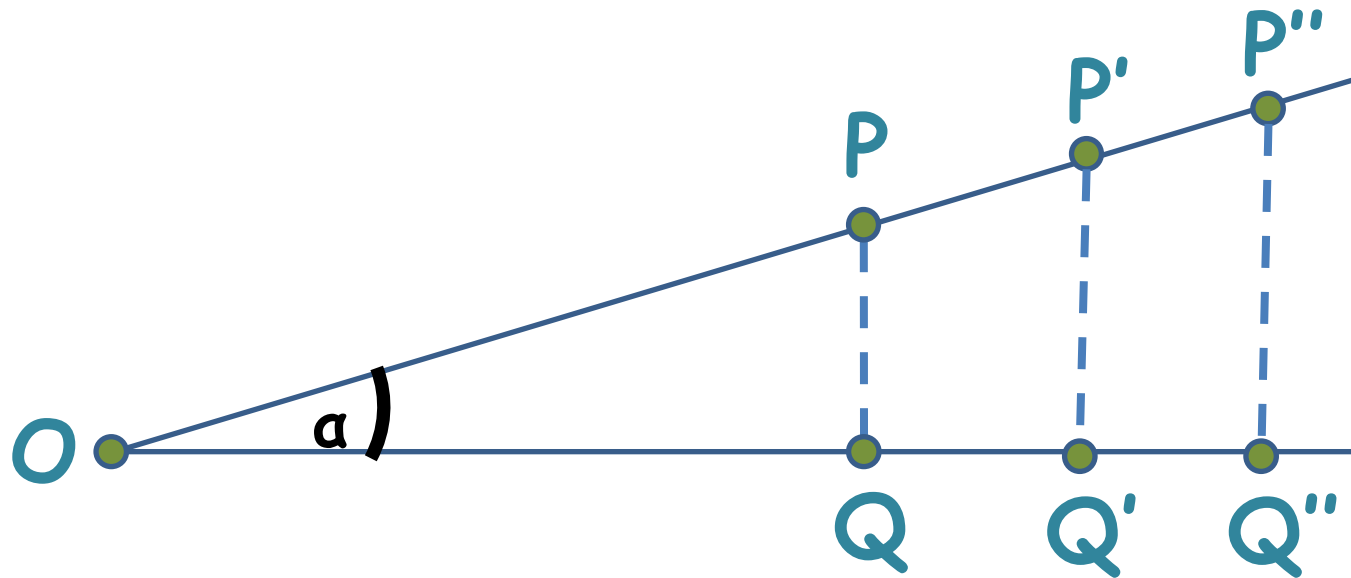
$$\rho = \frac{l}{r}$$

$$\alpha : 360^\circ = \rho : 2\pi$$

Le funzioni goniometriche

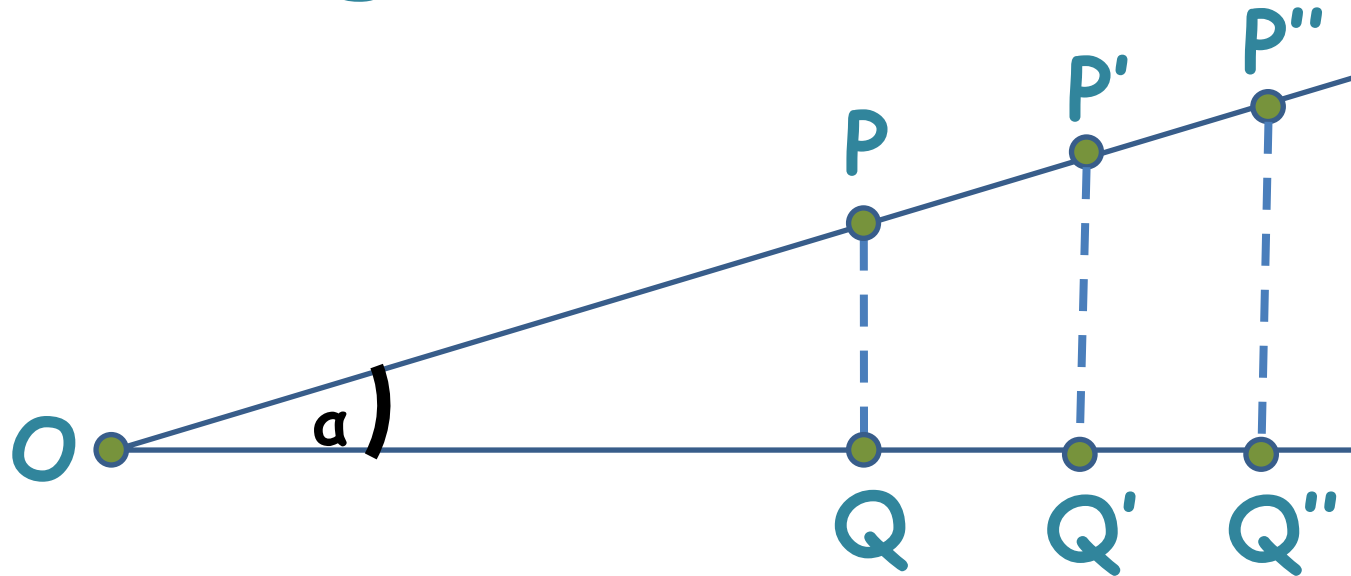


Le funzioni goniometriche



$$\frac{PQ}{OP} = \frac{P'Q'}{OP'} = \frac{P''Q''}{OP''} = \sin \alpha$$

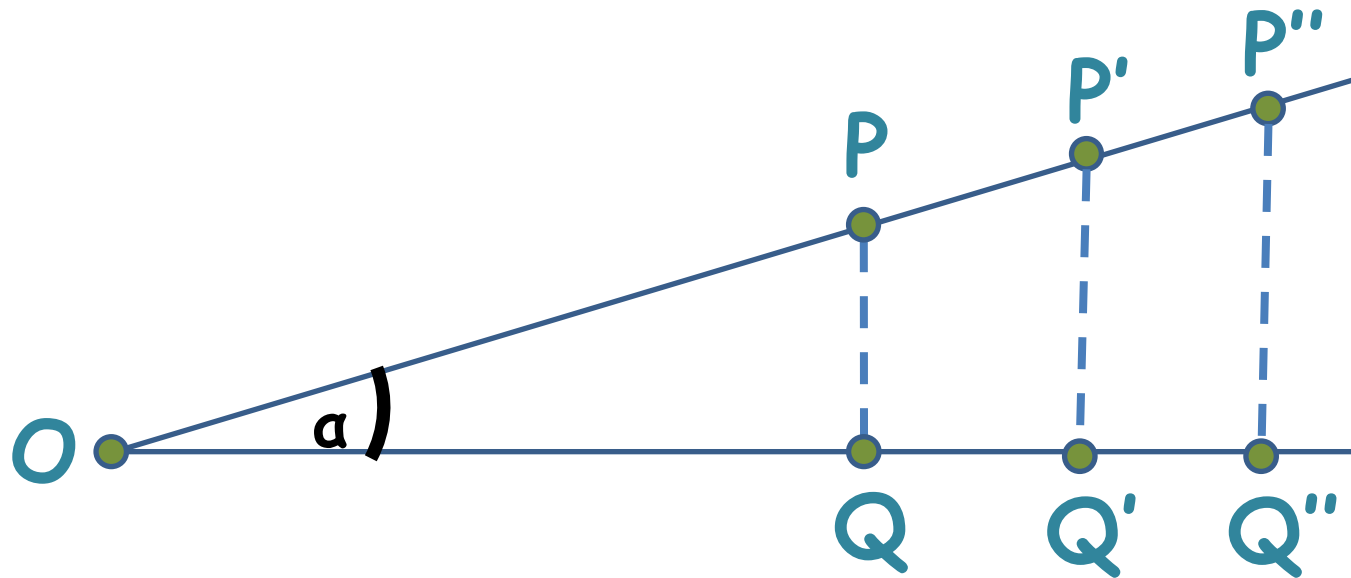
Le funzioni goniometriche



$$\frac{OQ}{OP} = \frac{OQ'}{OP'} = \frac{OQ''}{OP''} = \cos \alpha$$

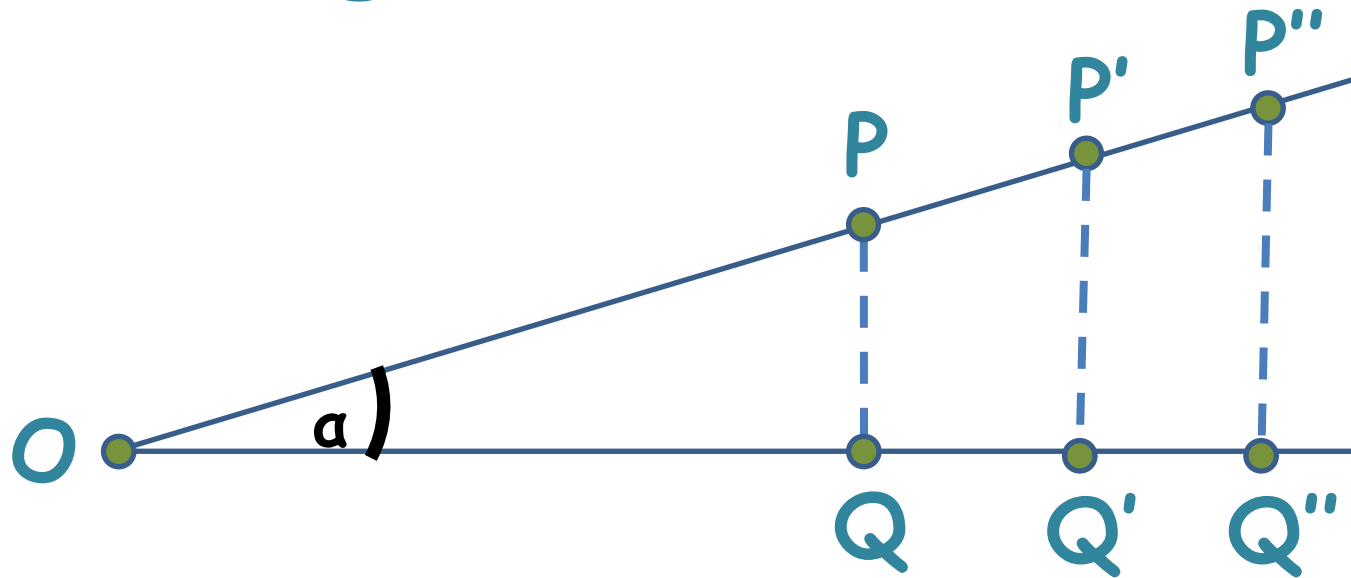
$$\cos^2 \alpha + \sin^2 \alpha = 1$$

Le funzioni goniometriche



$$\frac{PQ}{OQ} = \frac{P'Q'}{OQ'} = \frac{P''Q''}{OQ''} = \tan \alpha = \operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha}$$

Le funzioni goniometriche

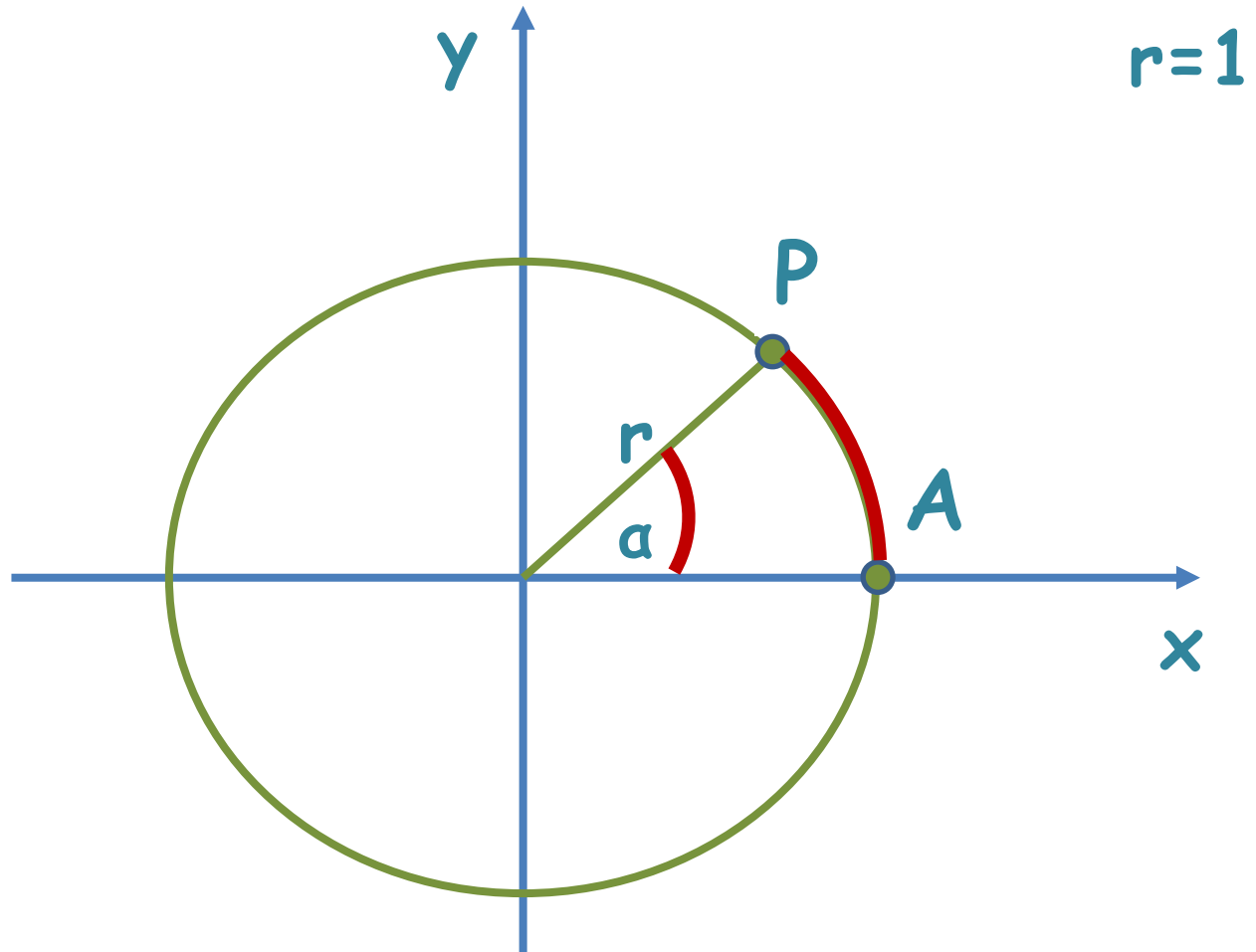


$$\frac{OP}{OQ} = \sec \alpha = \frac{1}{\cos \alpha}$$

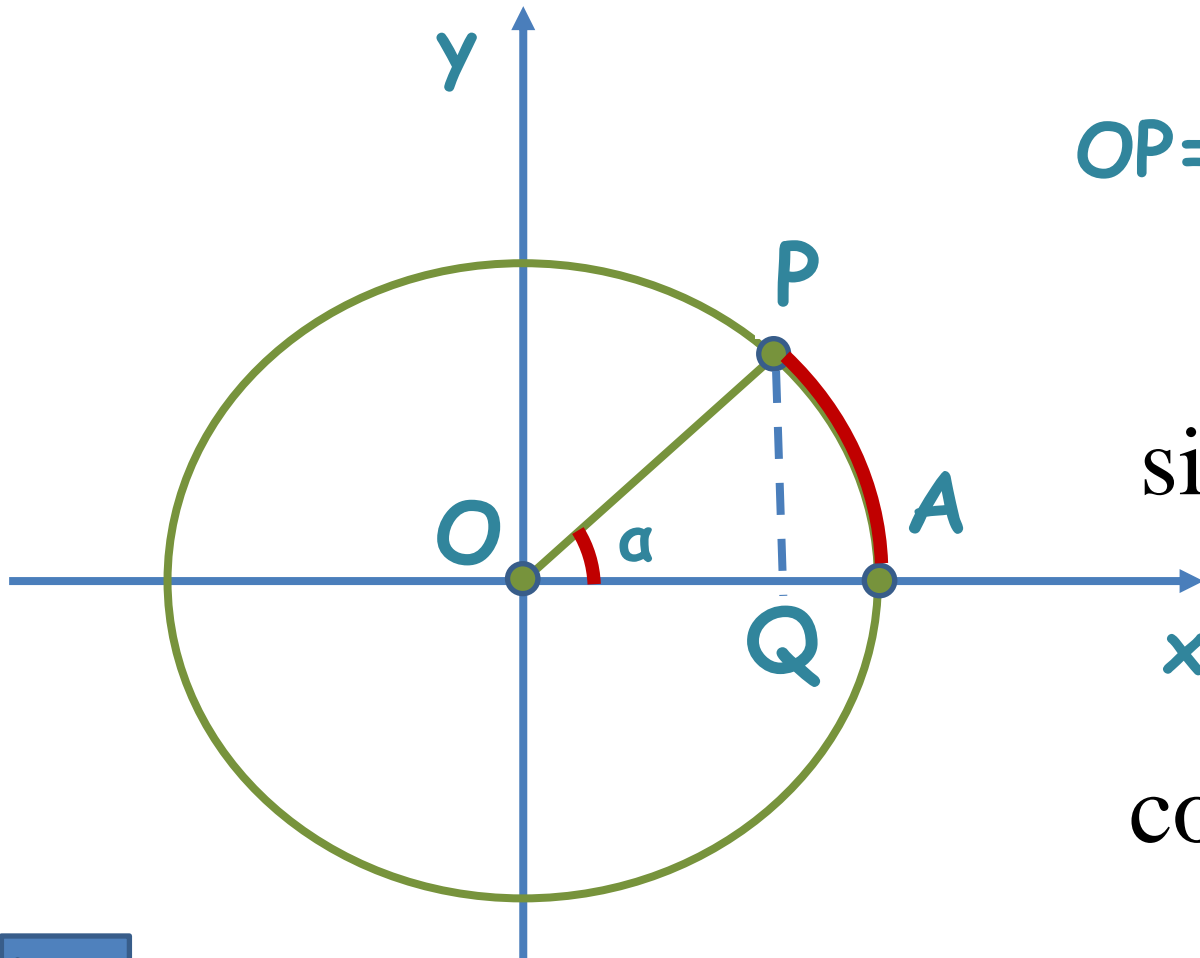
$$\frac{OP}{PQ} = \operatorname{cosec} \alpha = \frac{1}{\sin \alpha}$$

$$\frac{OQ}{PQ} = \operatorname{cotg} \alpha = \frac{1}{\operatorname{tg} \alpha}$$

La circonferenza goniometrica



Le funzioni trigonometriche



$$OP=r=1$$

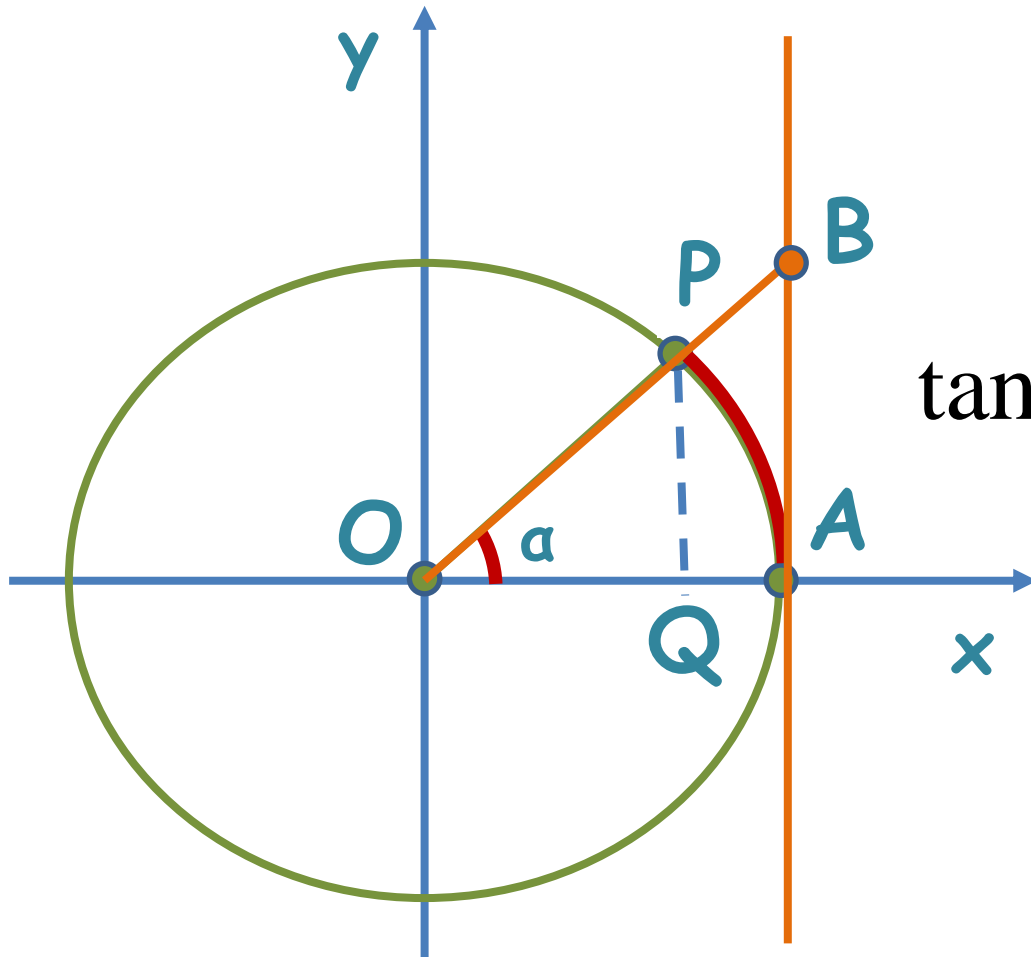
$$\sin \alpha = \frac{PQ}{OP} = PQ$$

$$\cos \alpha = \frac{OQ}{OP} = OQ$$

$$\cos^2 \alpha + \sin^2 \alpha = 1$$



Le funzioni trigonometriche

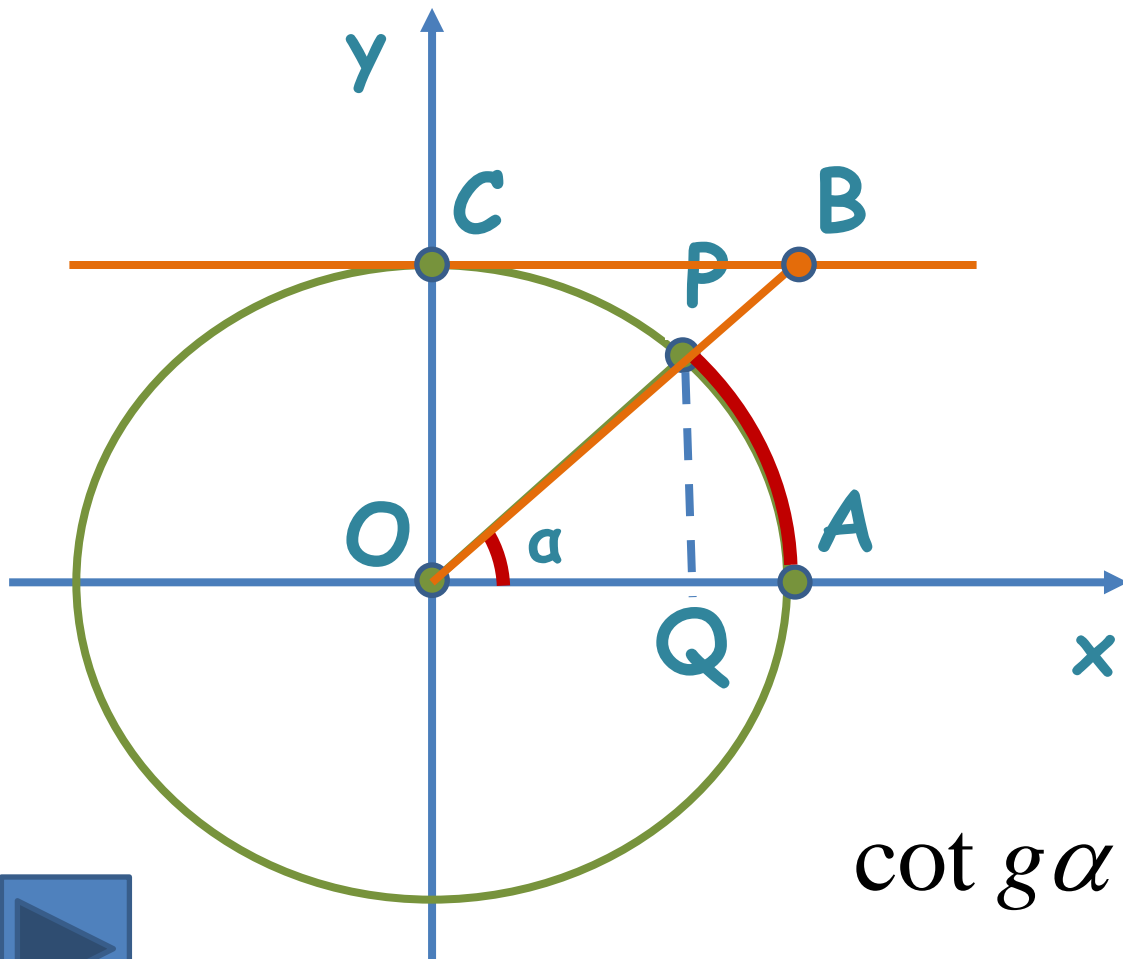


$$OP=OA=r=1$$

$$\tan \alpha = \frac{PQ}{OQ} = \frac{BA}{OA} = BA$$

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha} \neq 0$$

Le funzioni trigonometriche

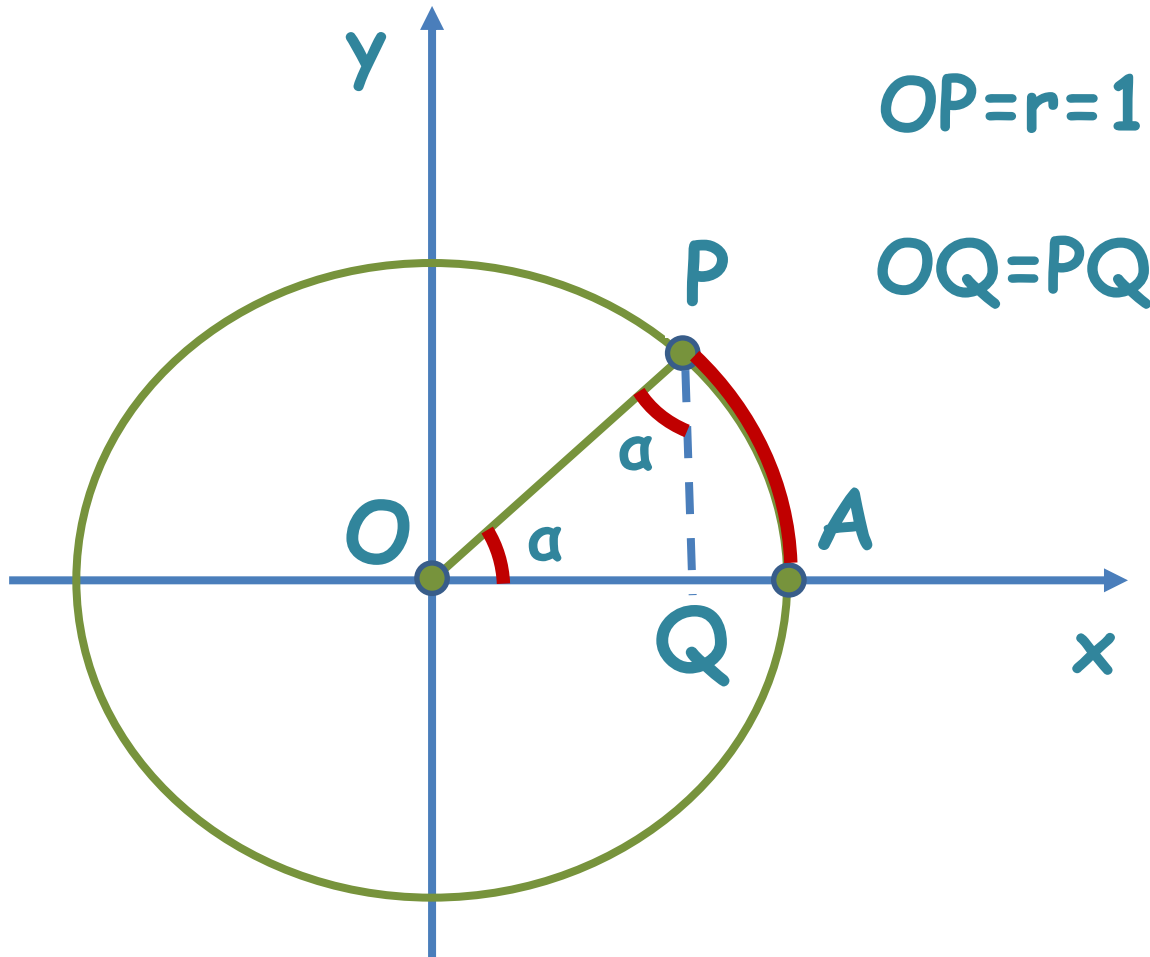


$$OP=OC=r=1$$

$$\cot g \alpha = \frac{OQ}{PQ} = \frac{CB}{OC} = CB$$



Angoli fondamentali



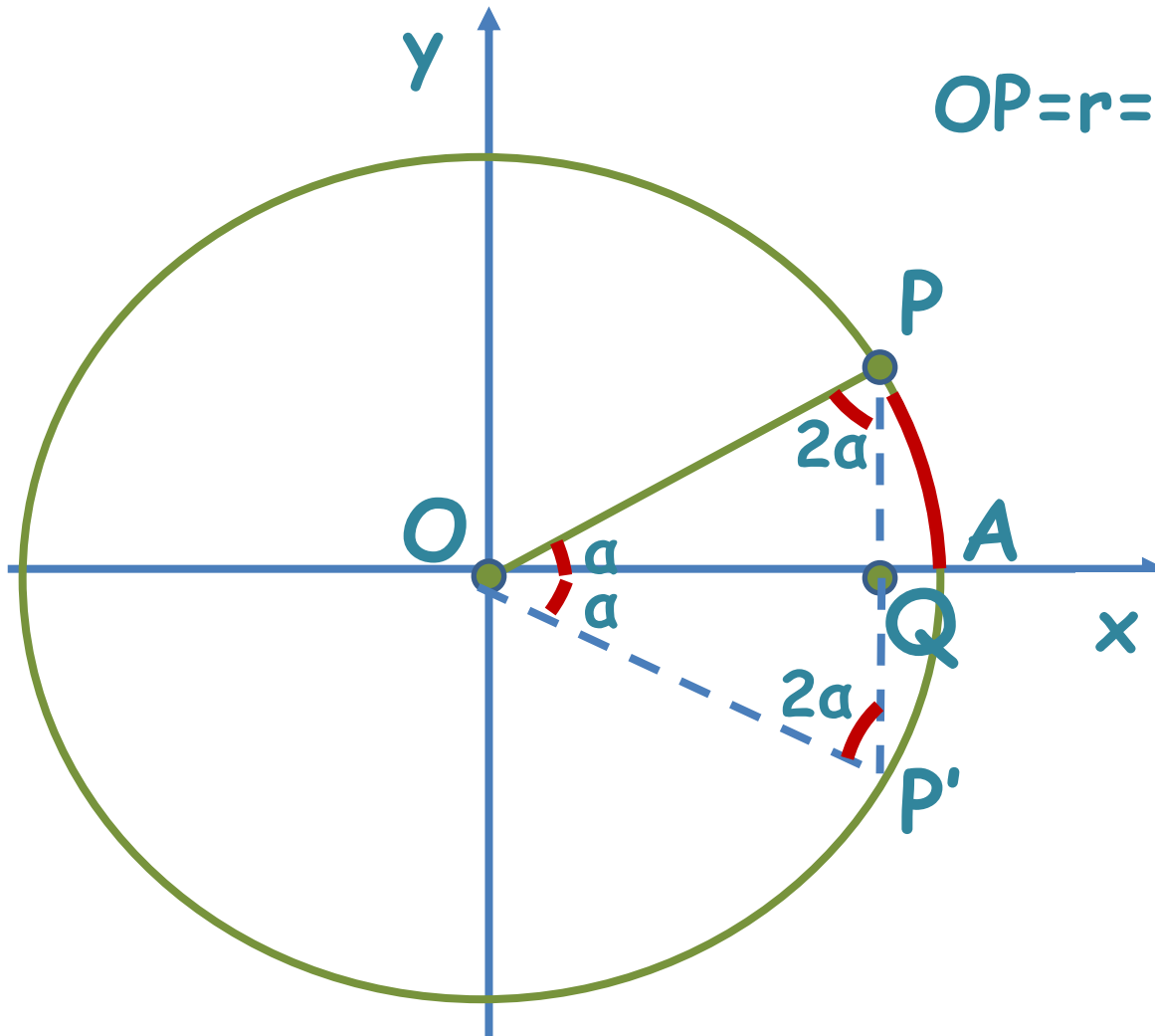
$$\alpha = 45^\circ = \pi/4$$

$$\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\tan \frac{\pi}{4} = \frac{PQ}{OQ} = 1$$

Angoli fondamentali



$$OP=r=1$$

$$\alpha=30^\circ=\pi/6$$

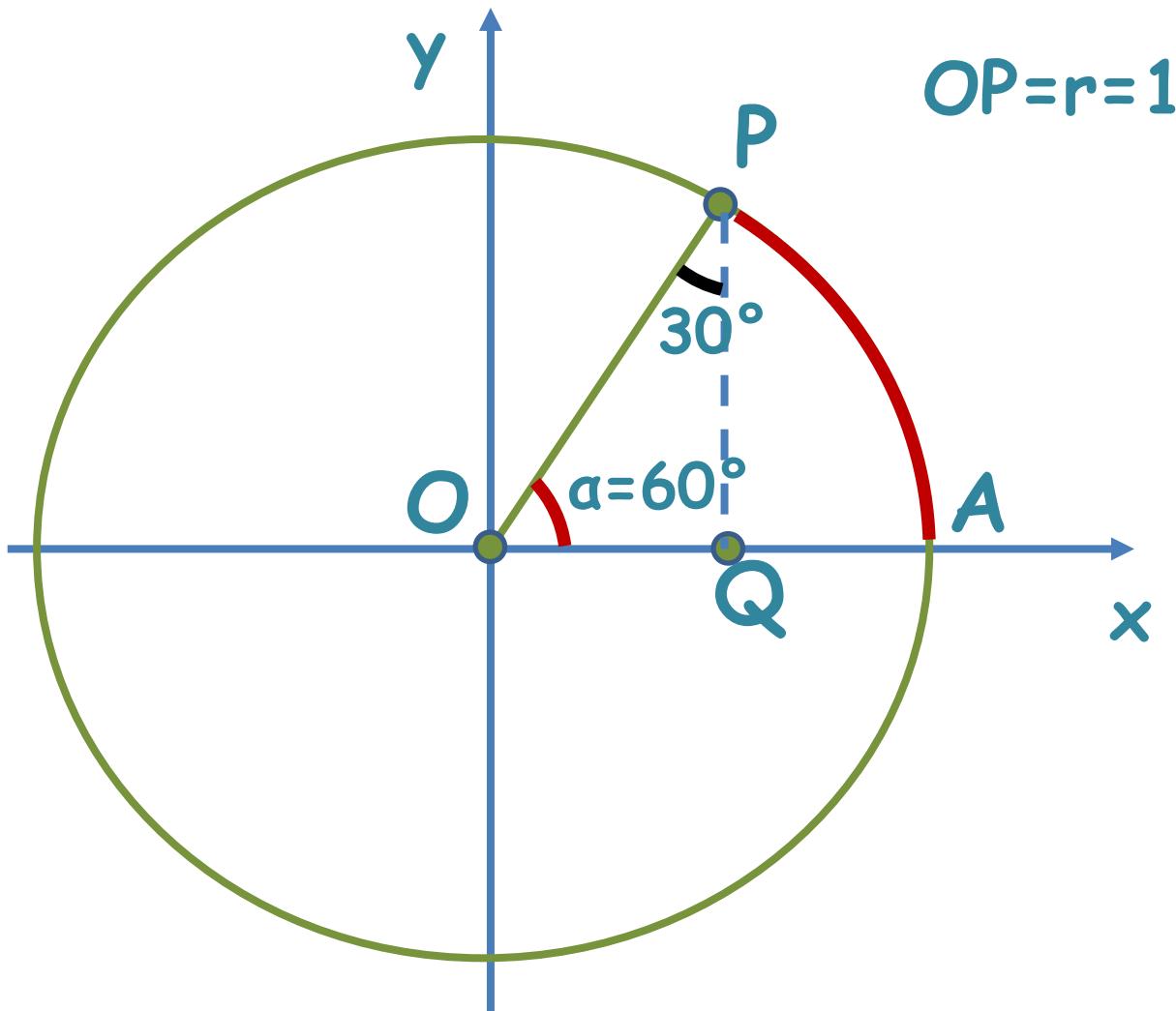
$$OP=PP'=OP'$$

$$\sin \frac{\pi}{6} = \frac{1}{2}$$

$$\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$\tan \frac{\pi}{6} = \frac{\sqrt{3}}{3}$$

Angoli fondamentali



$$OP=r=1$$

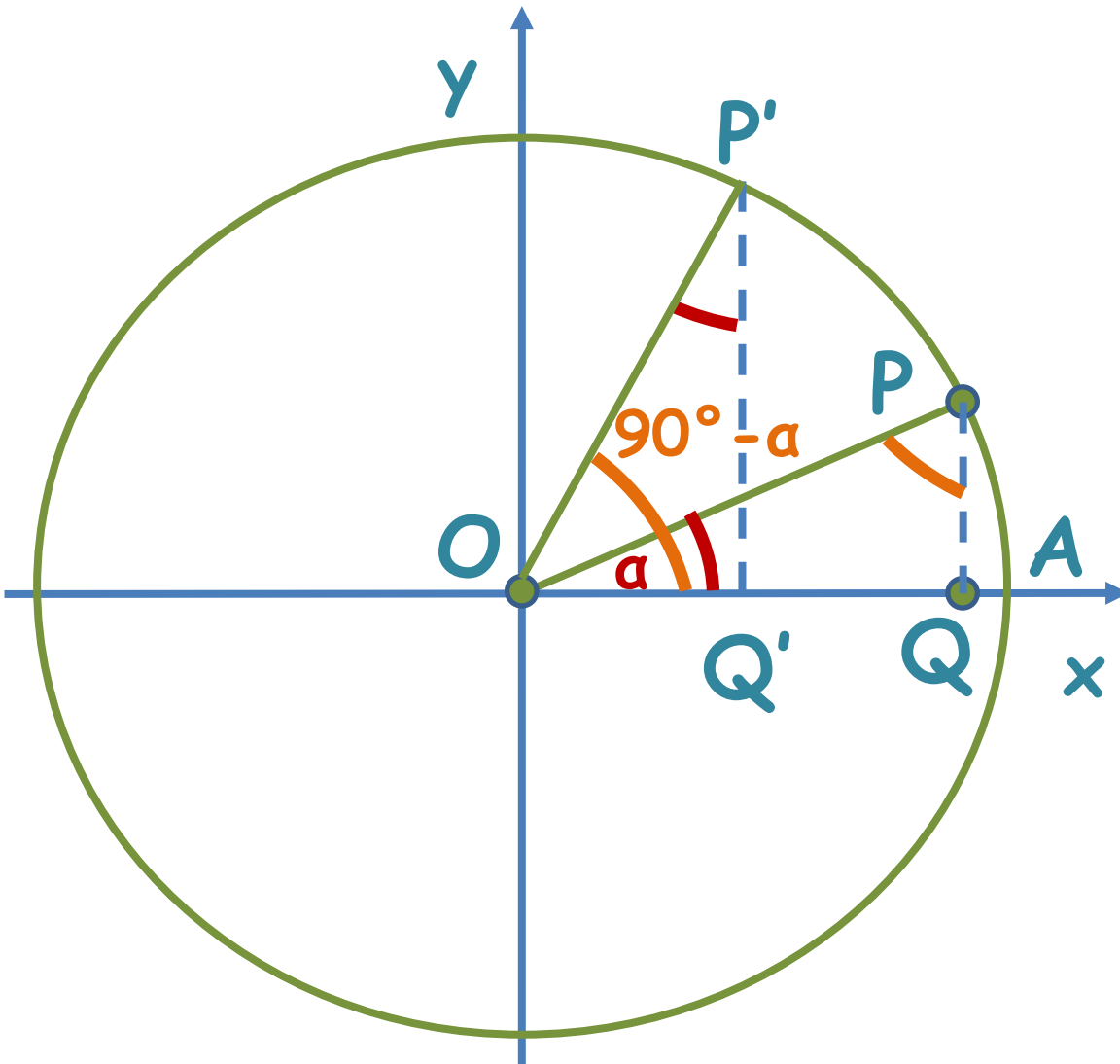
$$\alpha=60^\circ = \pi/3$$

$$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\cos \frac{\pi}{3} = \frac{1}{2}$$

$$\tan \frac{\pi}{3} = \sqrt{3}$$

Angoli complementari



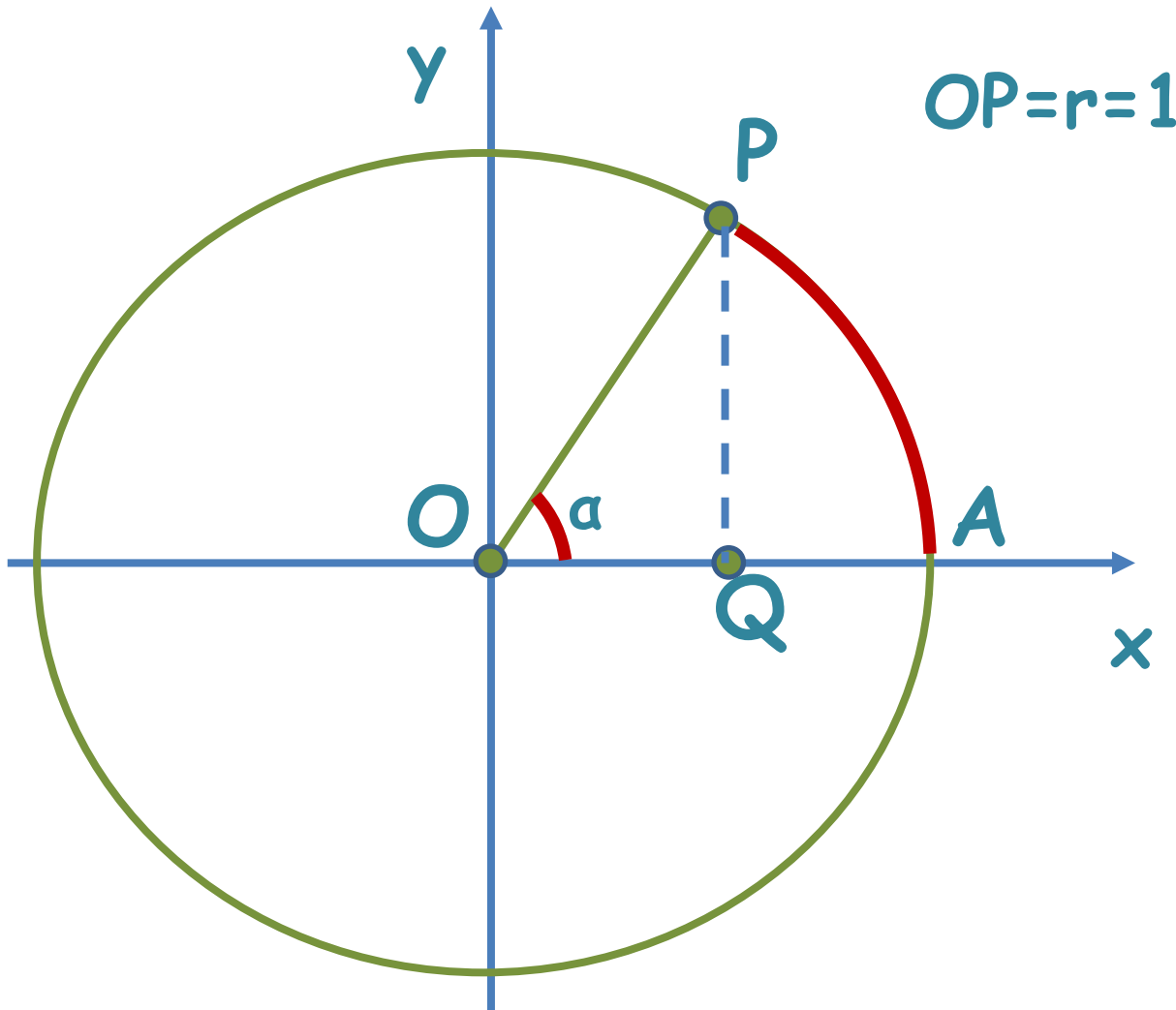
$$OP = OP' = r = 1$$

$$\sin \alpha = \cos \left(\frac{\pi}{2} - \alpha \right)$$

$$\cos \alpha = \sin \left(\frac{\pi}{2} - \alpha \right)$$

$$\tan \alpha = \cot \left(\frac{\pi}{2} - \alpha \right)$$

Angoli fondamentali



$$OP=r=1$$

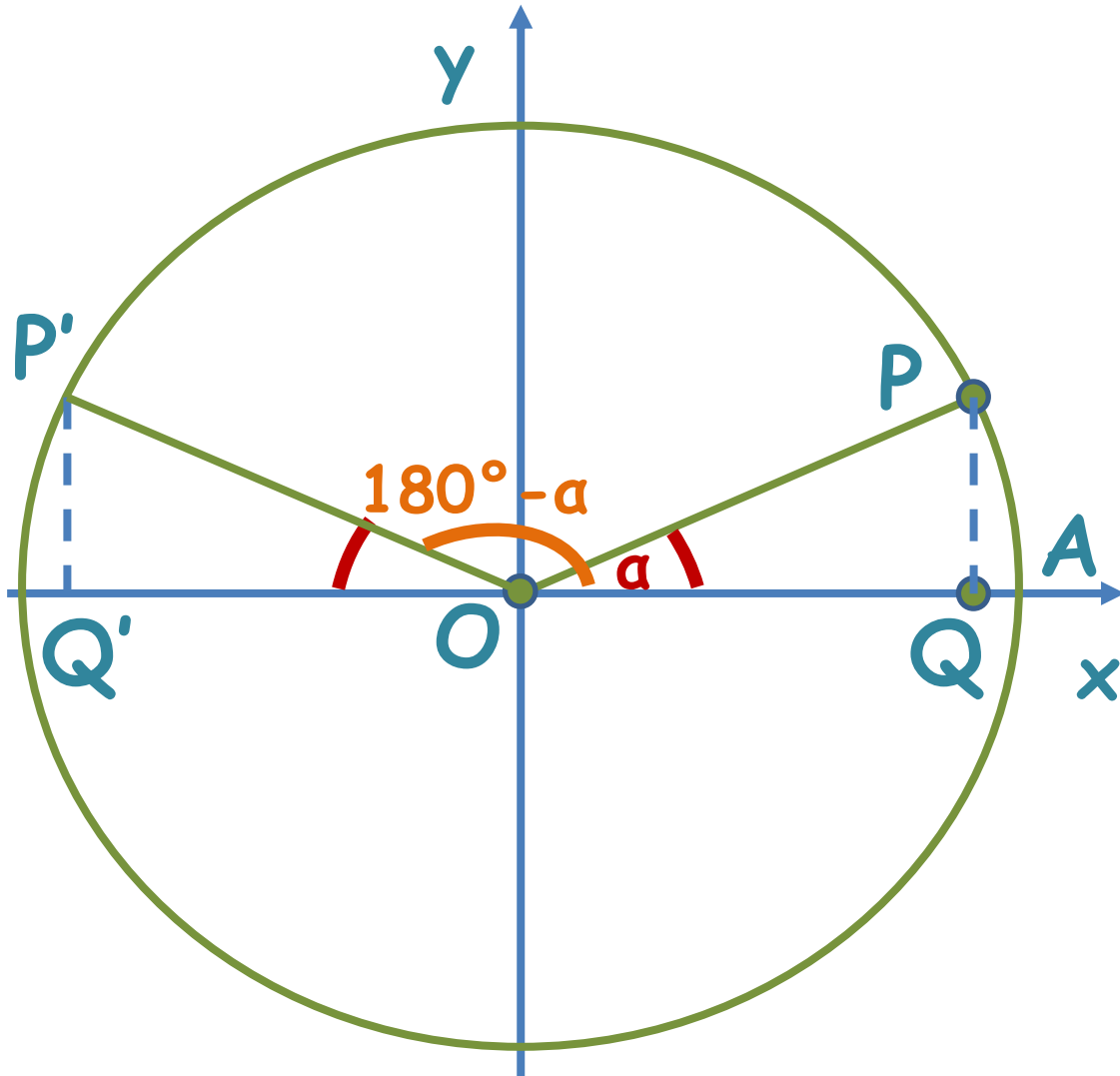
$$\alpha=60^\circ=\pi/3$$

$$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\cos \frac{\pi}{3} = \frac{1}{2}$$

$$\tan \frac{\pi}{3} = \sqrt{3}$$

Angoli supplementari



$$OP = OP' = r = 1$$

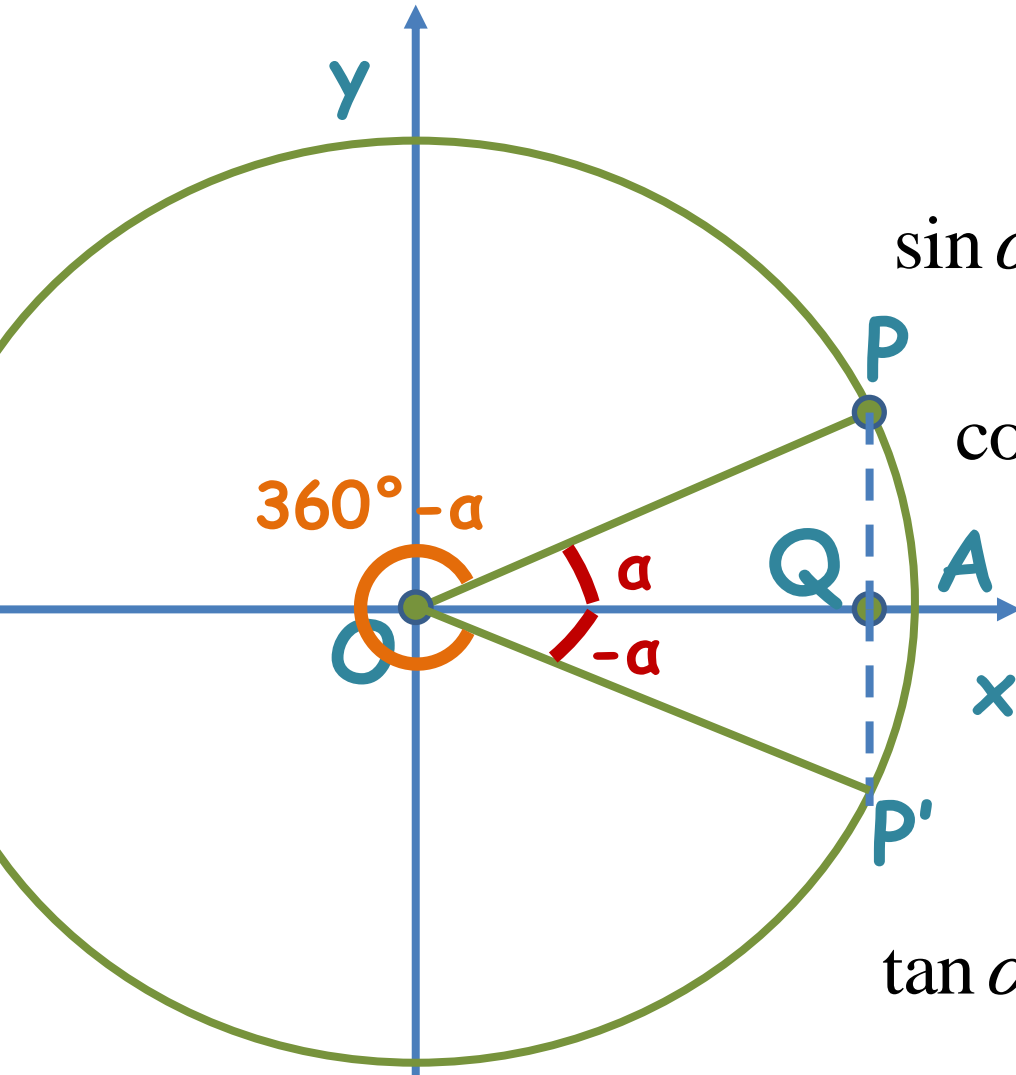
$$\sin \alpha = \sin(\pi - \alpha)$$

$$\cos \alpha = -\cos(\pi - \alpha)$$

$$\tan \alpha = -\tan(\pi - \alpha)$$

Angoli esplementari o opposti

$$OP=OP'=r=1$$

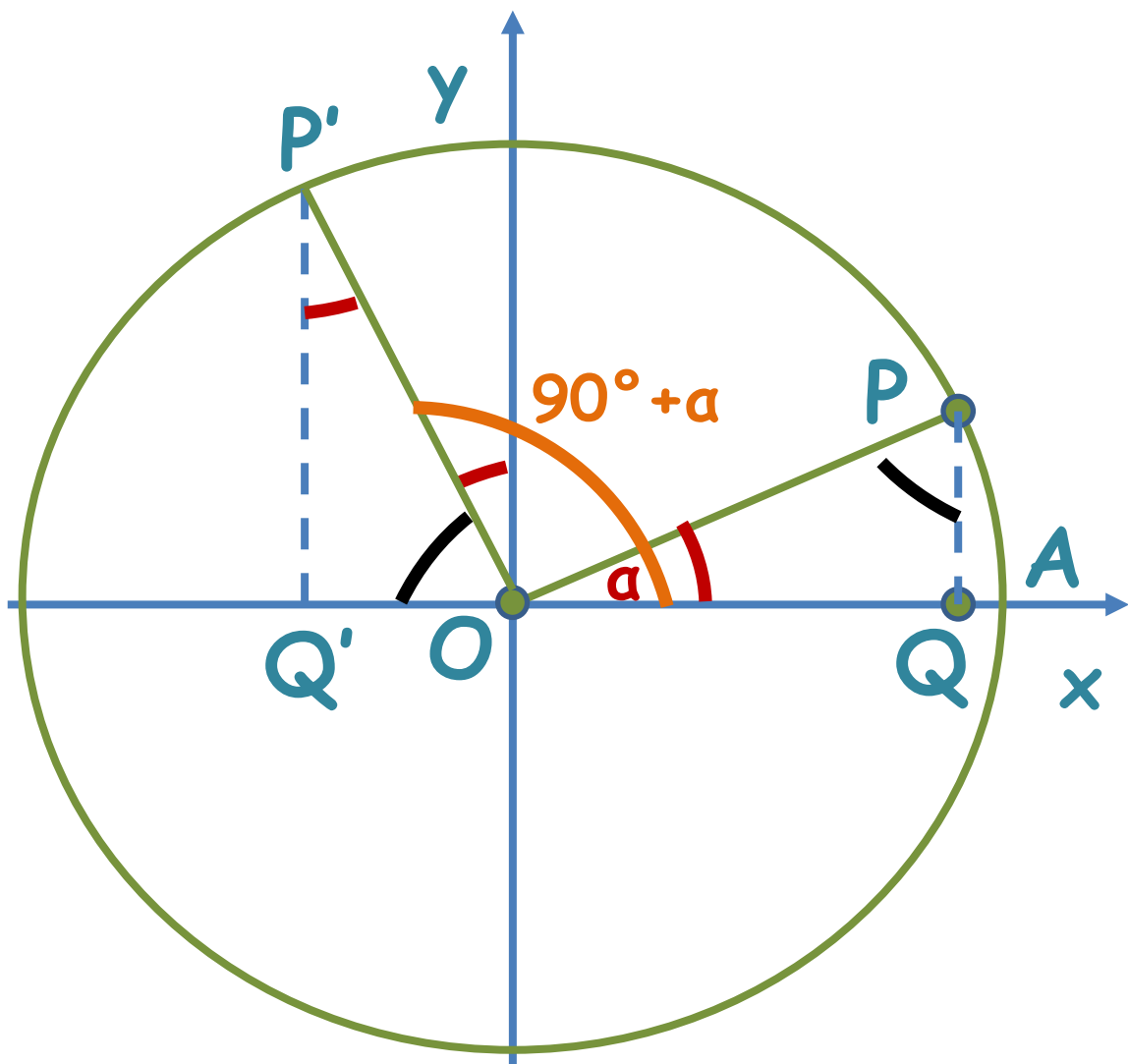


$$\sin \alpha = -\sin(2\pi - \alpha) = -\sin(-\alpha)$$

$$\cos \alpha = \cos(2\pi - \alpha) = \cos(-\alpha)$$

$$\tan \alpha = -\tan(2\pi - \alpha) = -\tan(-\alpha)$$

Angoli che differiscono di 90°



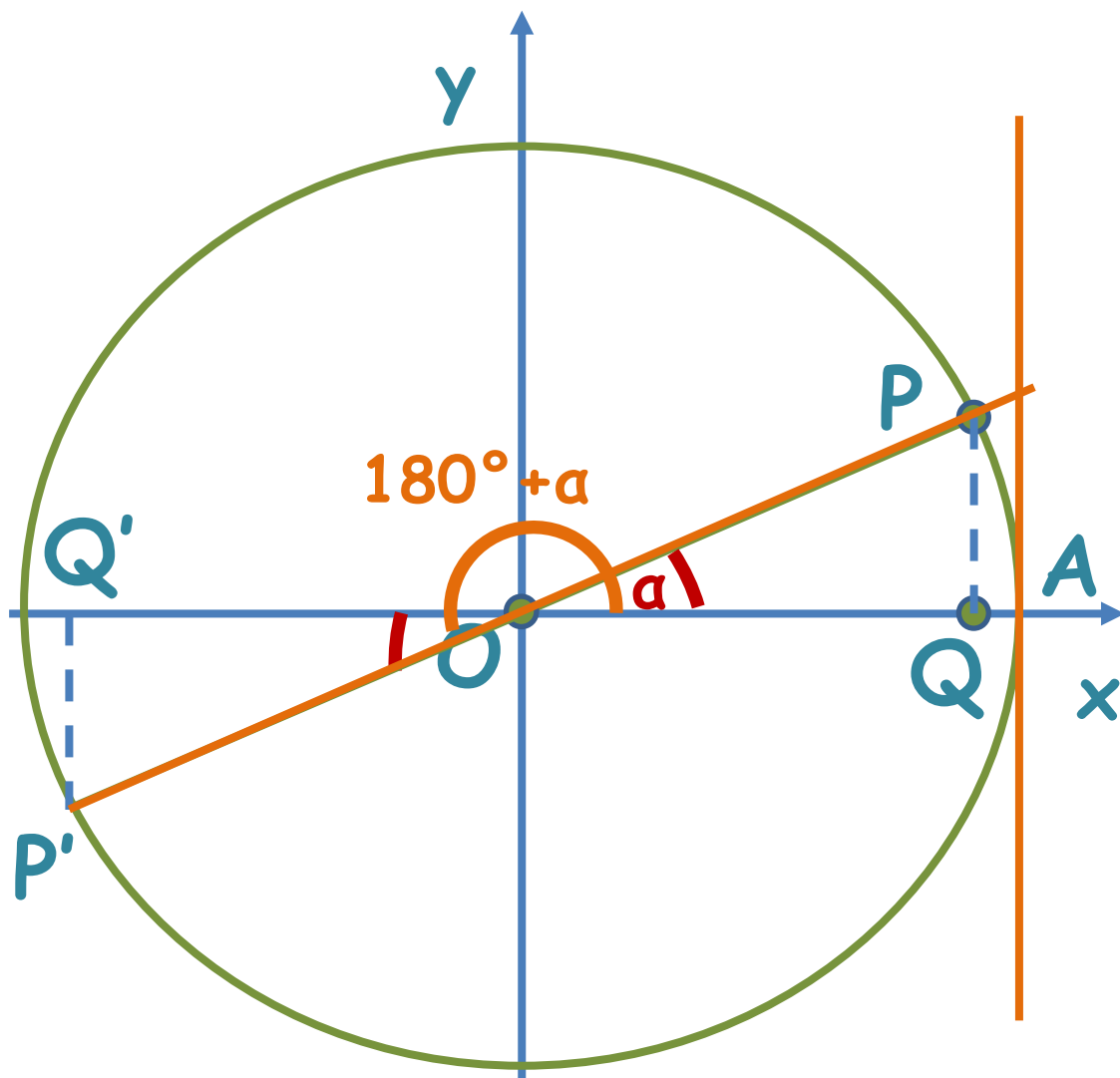
$$OP = OP' = r = 1$$

$$\sin \alpha = -\cos \left(\frac{\pi}{2} + \alpha \right)$$

$$\cos \alpha = \sin \left(\frac{\pi}{2} + \alpha \right)$$

$$\tan \alpha = -\cot \left(\frac{\pi}{2} + \alpha \right)$$

Angoli che differiscono di 180°



$$OP = OP' = r = 1$$

$$\sin \alpha = -\sin(\pi + \alpha)$$

$$\cos \alpha = -\cos(\pi + \alpha)$$

$$\tan \alpha = \tan(\pi + \alpha)$$

Sinusoid

α	$\sin \alpha$
0	0
$\pi/6$	1/2
$\pi/4$	$\sqrt{2}/2$
$\pi/3$	$\sqrt{3}/2$
$\pi/2$	1
$\pi/2 < \alpha < \pi$	$\sin (\pi/2+\alpha)=\sin (\pi/2-\alpha)$
π	0
$\pi < \alpha < 3\pi/2$	$\sin (\pi+\alpha)=-\sin \alpha$
$3\pi/2$	-1
$3\pi/2 < \alpha < 2\pi$	$\sin (2\pi-\alpha)=-\sin \alpha$



Cosinusoidi

α	$\cos \alpha$
0	1
$\pi/6$	$\sqrt{3}/2$
$\pi/4$	$\sqrt{2}/2$
$\pi/3$	$1/2$
$\pi/2$	0
$\pi/2 < \alpha < \pi$	$\cos(\pi/2 + \alpha) = -\cos(\pi/2 - \alpha)$
π	-1
$\pi < \alpha < 3\pi/2$	$\cos(\pi + \alpha) = -\cos \alpha$
$3\pi/2$	0
$3\pi/2 < \alpha < 2\pi$	$\cos(2\pi - \alpha) = \cos \alpha$

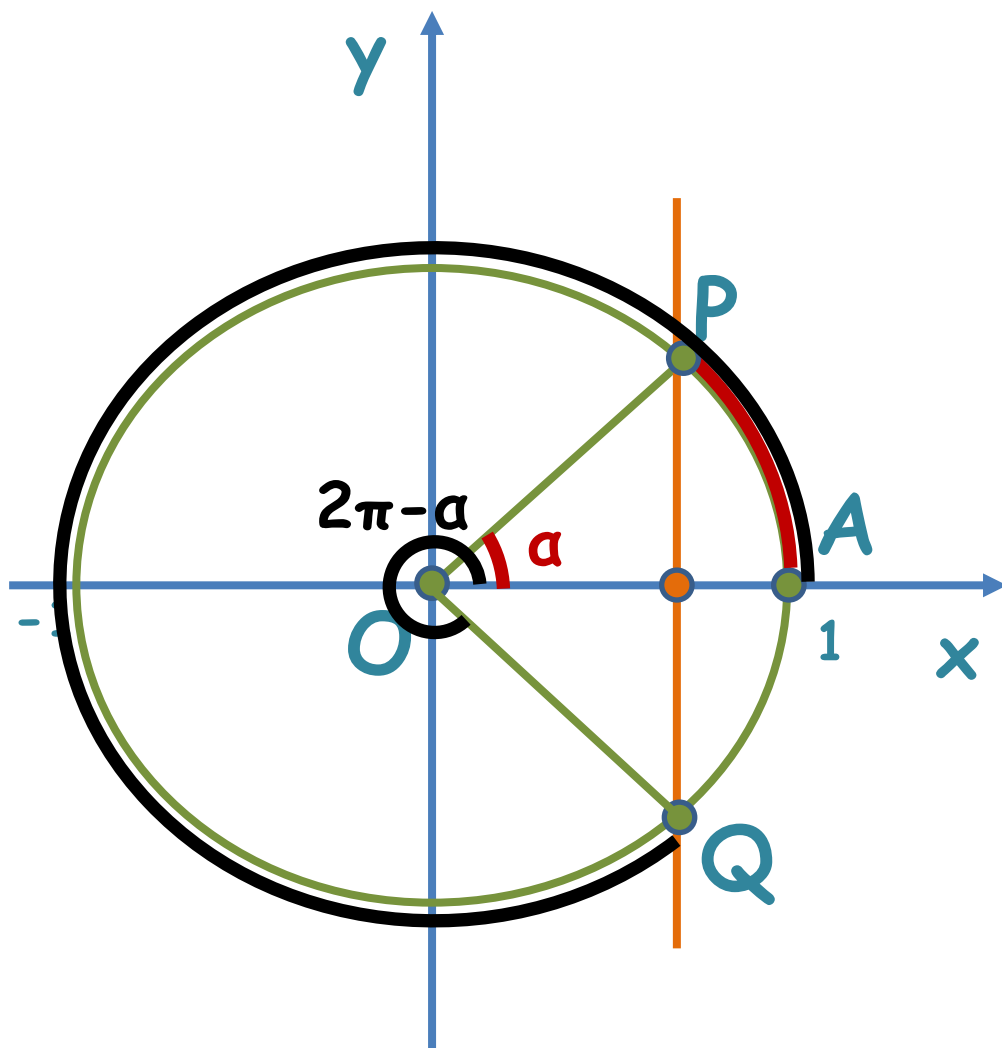


Tangente

α	$\tan \alpha$
0	0
$\pi/6$	$\sqrt{3}/3$
$\pi/4$	1
$\pi/3$	$\sqrt{3}$
$\pi/2$	Non definita
$\pi/2 < \alpha < \pi$	$\operatorname{tg}(\pi/2 + \alpha) = -\operatorname{tg}(\pi/2 - \alpha)$
π	0
$\pi < \alpha < 3\pi/2$	$\operatorname{tg}(\pi + \alpha) = \operatorname{tg} \alpha$
$3\pi/2$	Non definita
$3\pi/2 < \alpha < 2\pi$	$\operatorname{tg}(2\pi - \alpha) = -\operatorname{tg} \alpha$



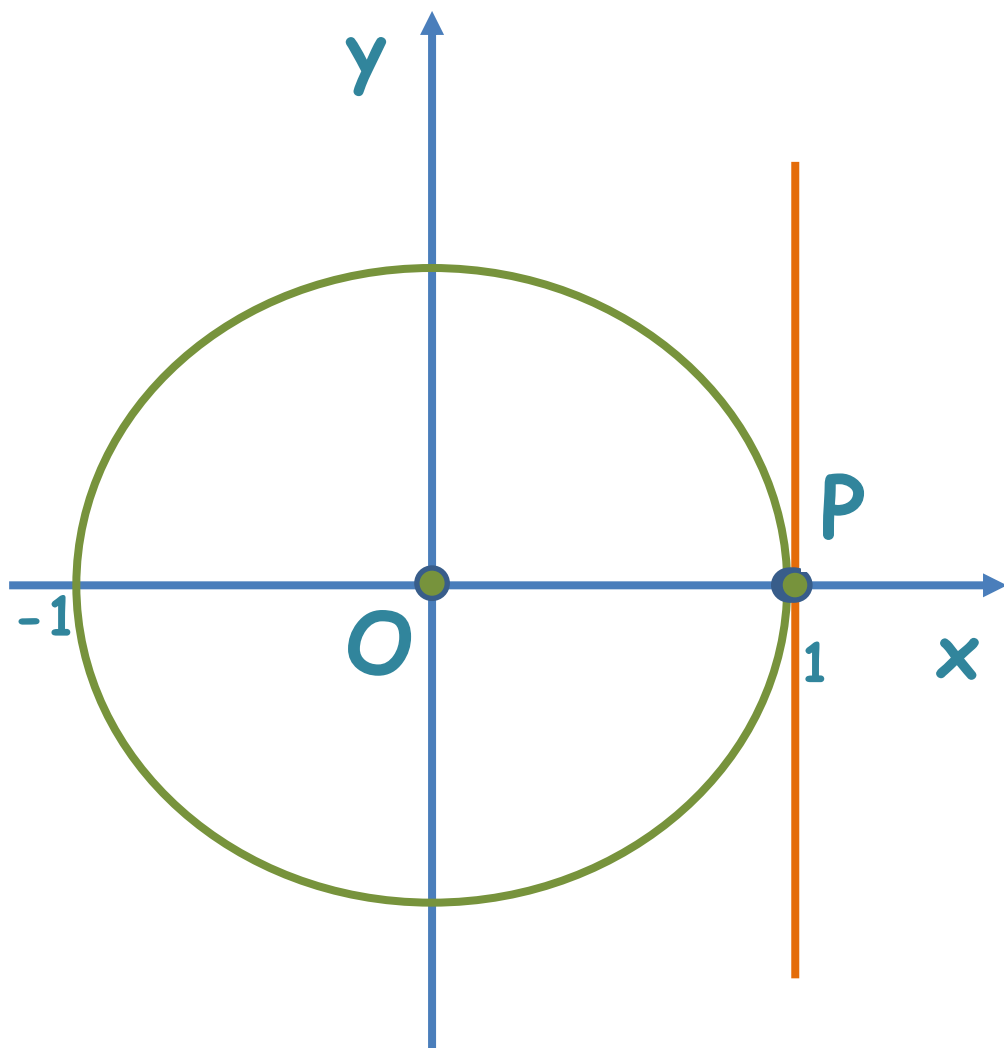
Equazioni trigonometriche elementari



$$\cos \alpha = q$$

$-1 < q < 1$ 2 soluzioni:
 $\alpha, 2\pi - \alpha$ ($-\alpha$)

Equazioni trigonometriche elementari

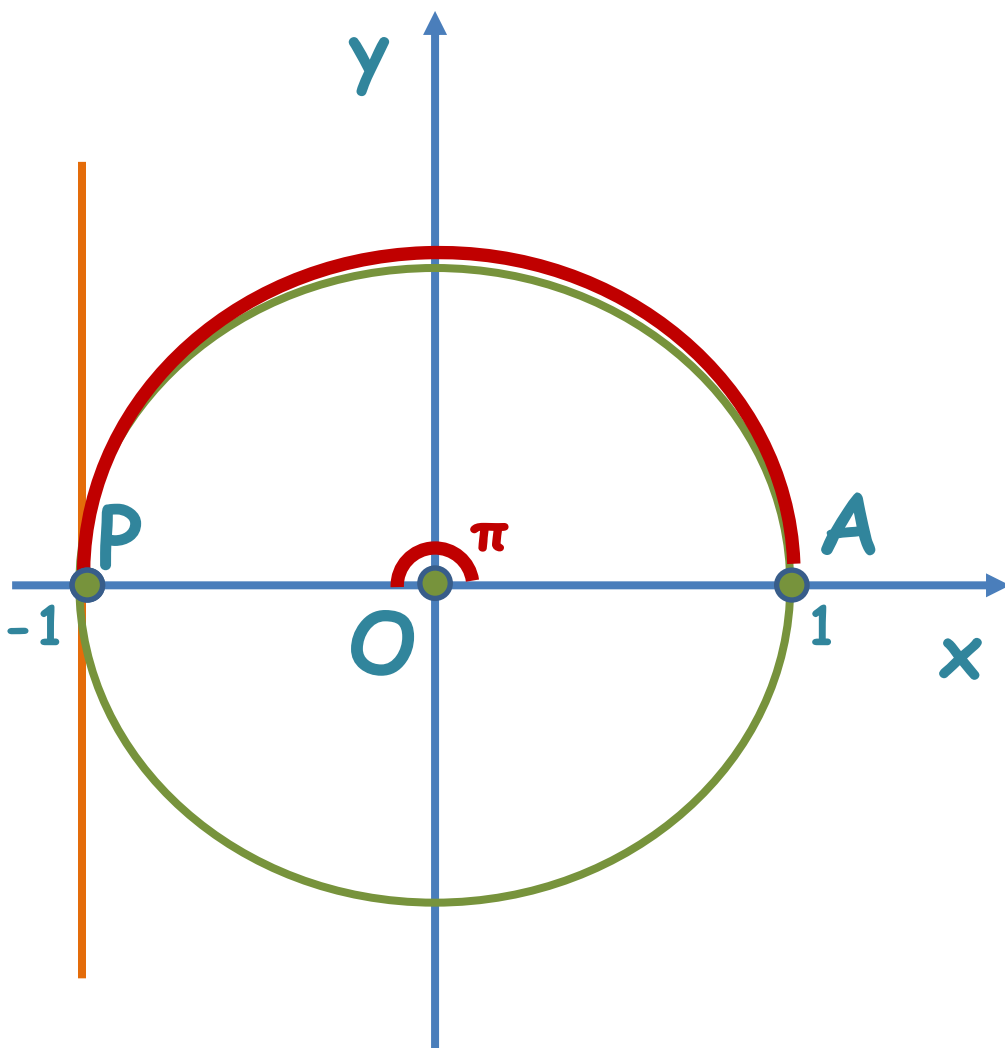


$$\cos \alpha = q$$

$-1 < q < 1$ 2 soluzioni:
 $\alpha, -\alpha$

$q = 1$ 1 soluzione: 0

Equazioni trigonometriche elementari



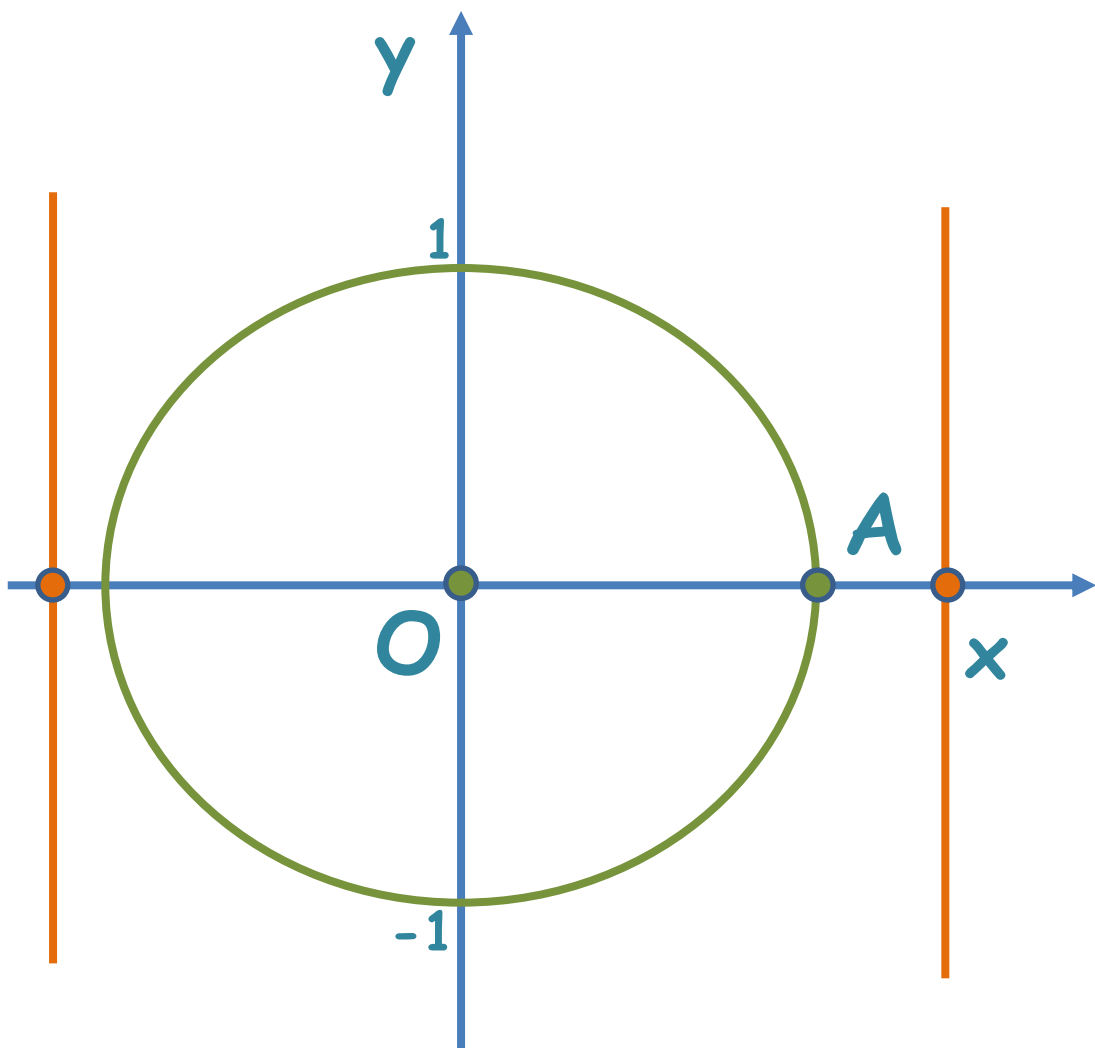
$$\cos \alpha = q$$

$-1 < q < 1$ 2 soluzioni:
 $\alpha, \pi - \alpha$

$q = 1$ 1 soluzione: 0

$q = -1$ 1 soluzione: π

Equazioni trigonometriche elementari



$$\cos \alpha = q$$

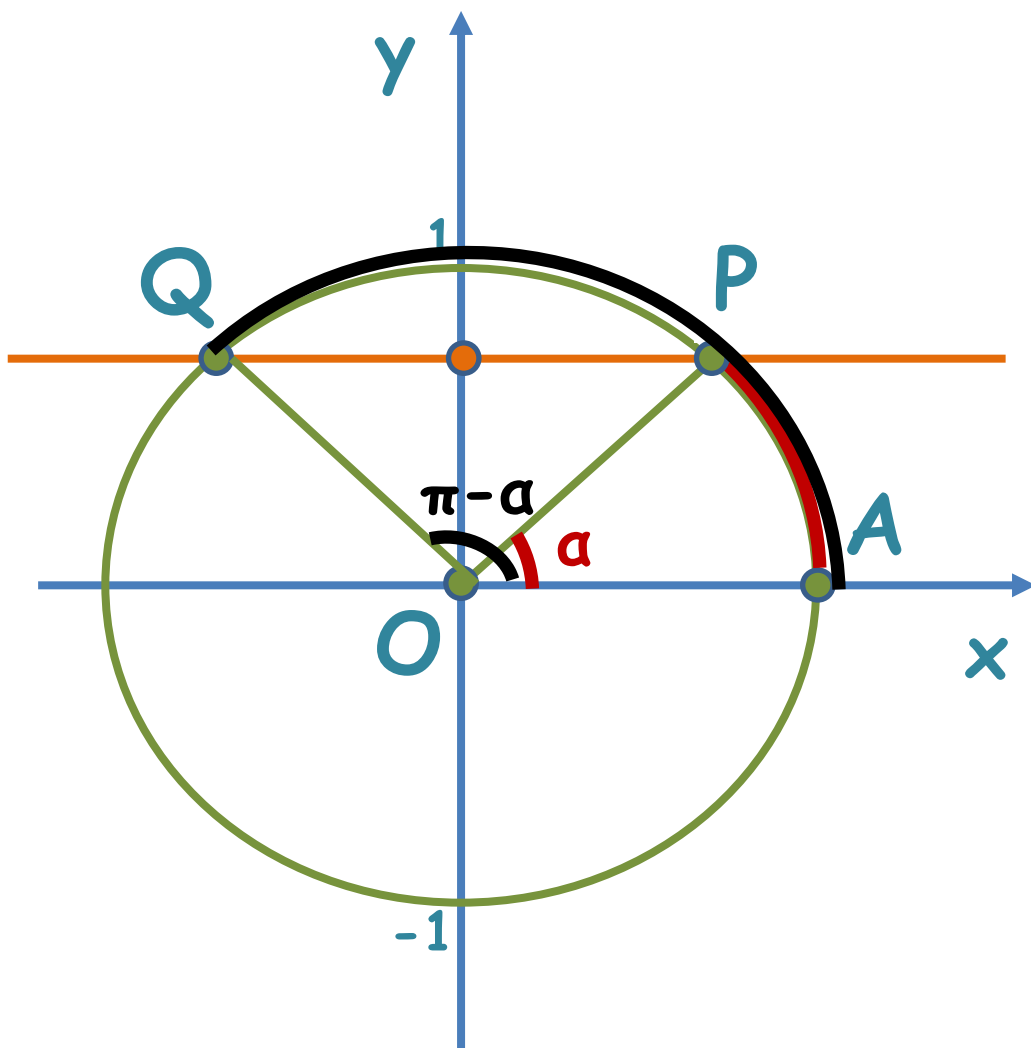
$-1 < q < 1$ 2 soluzioni:
 $\alpha, -\alpha$

$q = 1$ 1 soluzione: 0

$q = -1$ 1 soluzione: π

$q > 1$ }
 $q < -1$ } Nessuna
soluzione

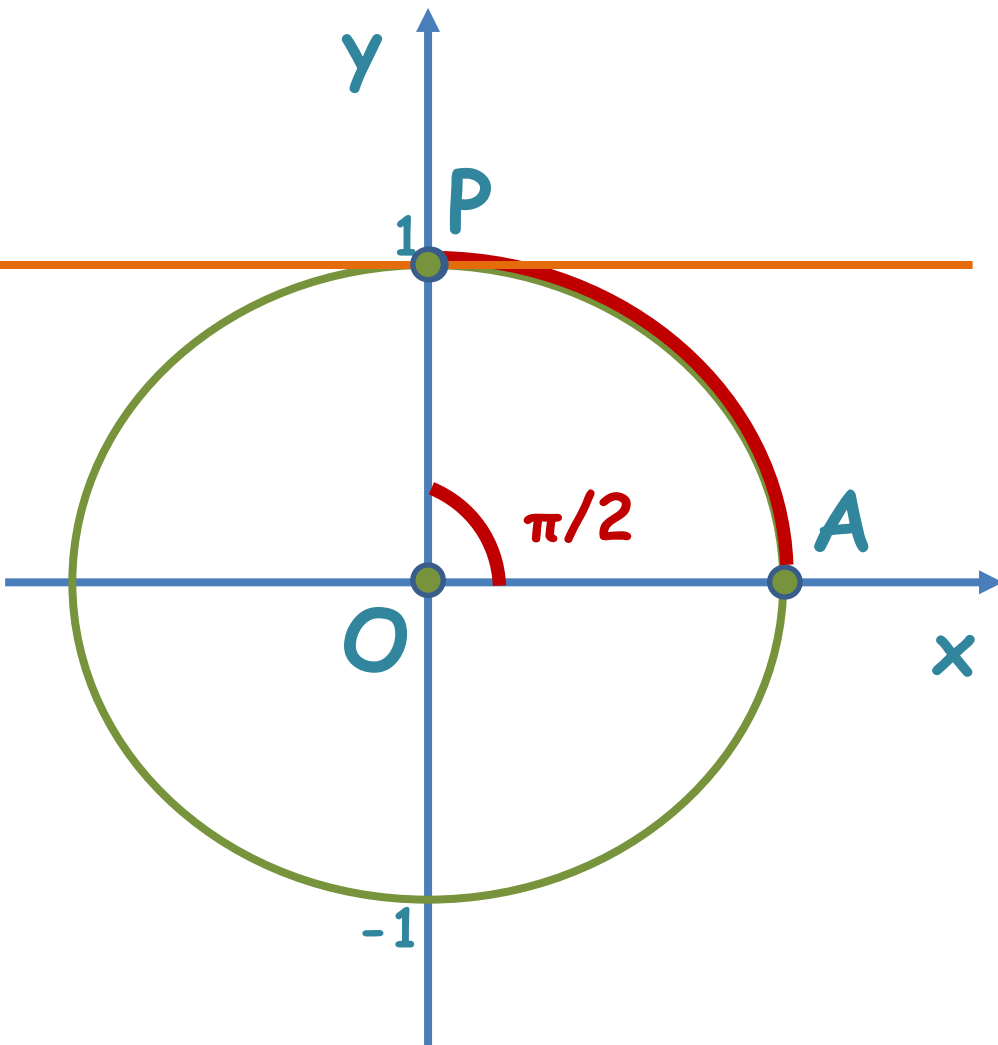
Equazioni trigonometriche elementari



$$\sin \alpha = p$$

$-1 < p < 1$ 2 soluzioni:
 $\alpha, \pi - \alpha$

Equazioni trigonometriche elementari

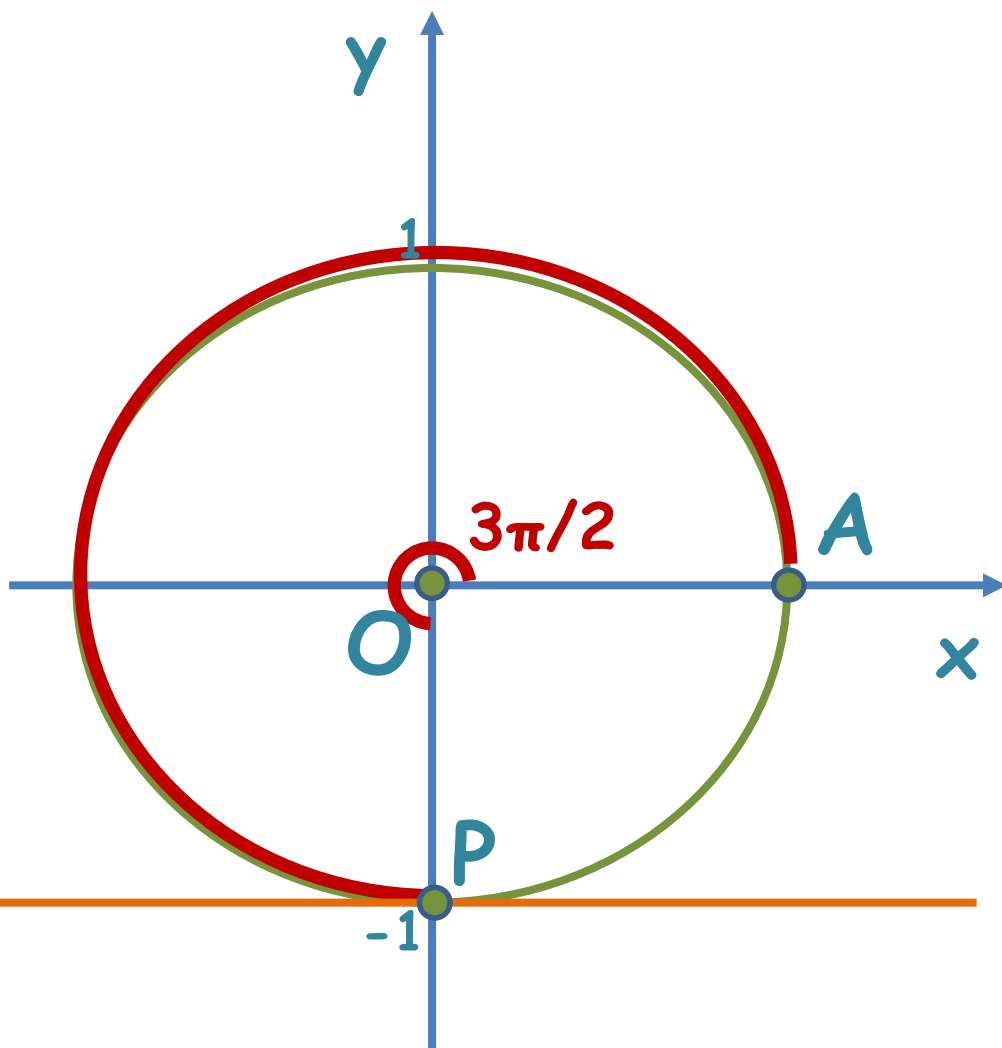


$$\sin \alpha = p$$

$-1 < p < 1$ 2 soluzioni:
 $\alpha, \pi - \alpha$

$p = 1$ 1 soluzione:
 $\pi/2$

Equazioni trigonometriche elementari



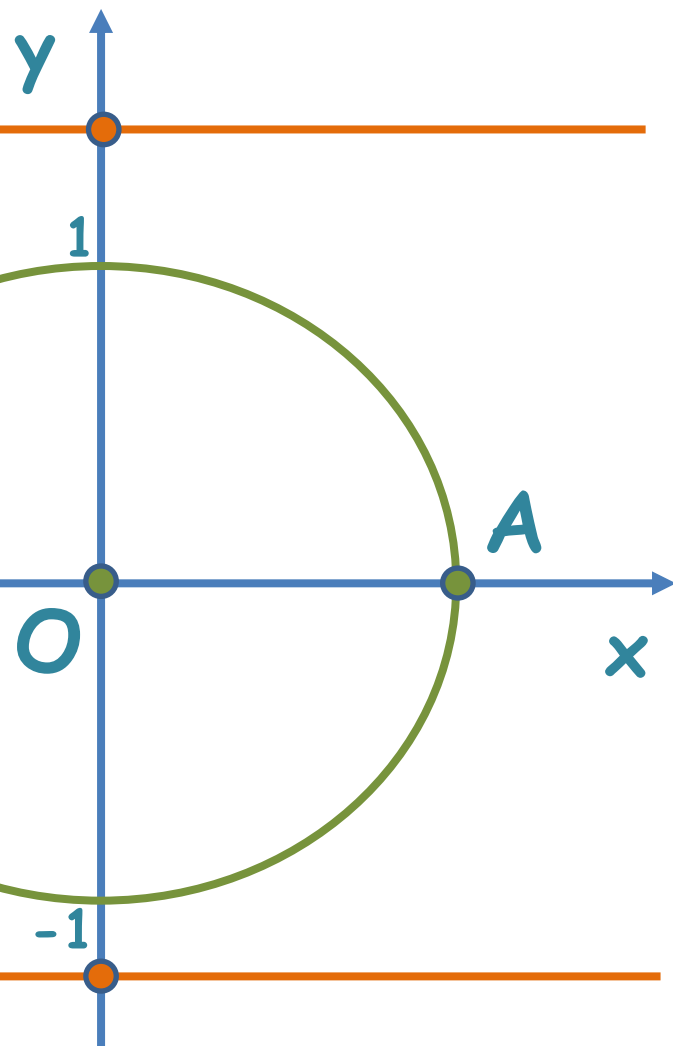
$$\sin \alpha = p$$

$-1 < p < 1$ 2 soluzioni:
 $\alpha, \pi - \alpha$

$p = 1$ 1 soluzione:
 $\pi/2$

$p = -1$ 1 soluzione:
 $3\pi/2$

Equazioni trigonometriche elementari



$$\sin \alpha = p$$

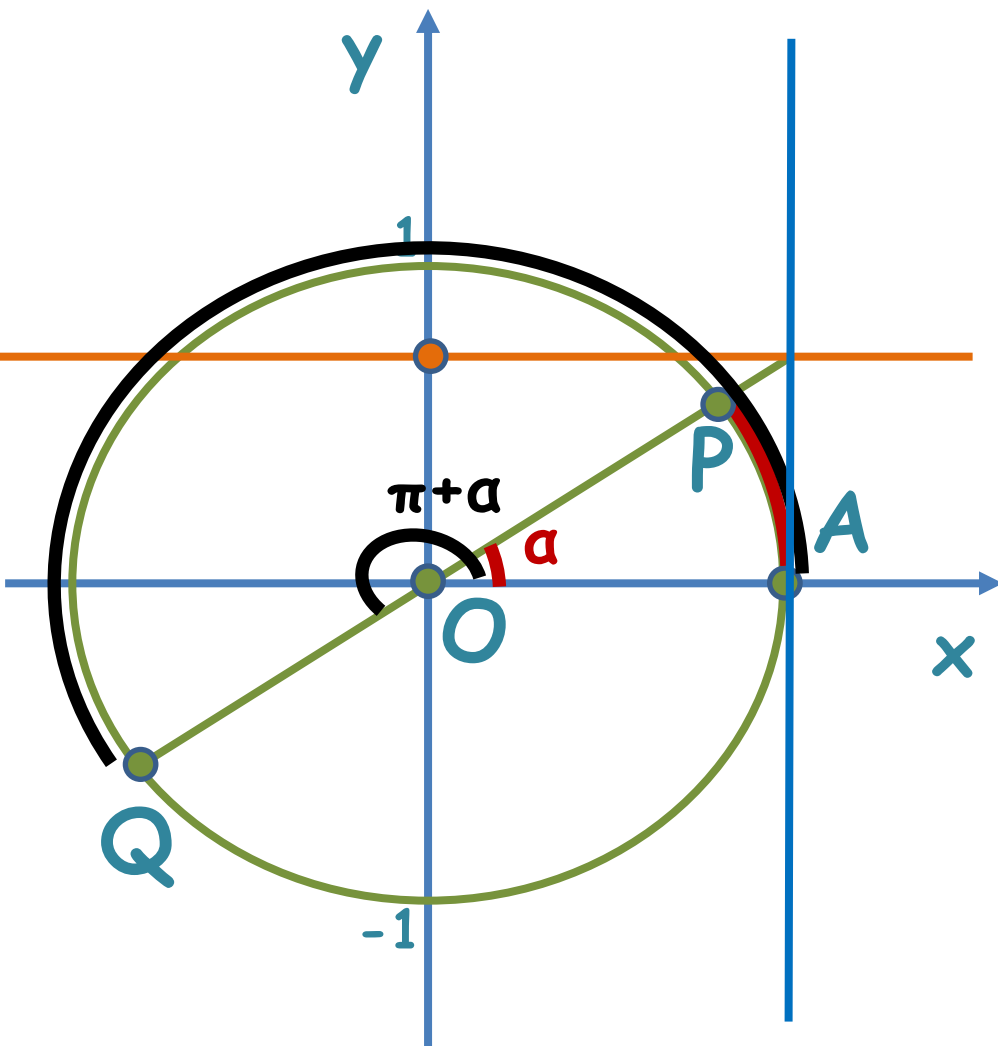
$-1 < p < 1$ 2 soluzioni:
 $\alpha, \pi - \alpha$

$p = 1$ 1 soluzione:
 $\pi/2$

$p = -1$ 1 soluzione:
 $3\pi/2$

$p > 1$ }
 $p < -1$ } Nessuna
soluzione

Equazioni trigonometriche elementari



$$\operatorname{tg} \alpha = m$$

$\forall m \in \mathbb{R}$ 2 soluzioni:
 $\alpha, \pi + \alpha$

ATTENZIONE:

$$\alpha \neq \pi/2$$
$$\alpha \neq 3\pi/2$$

Equazioni trigonometriche elementari

$$\sin x = \frac{1}{2}$$

$$\sin x = -1$$

$$\sin 2x = \frac{\sqrt{2}}{2}$$

$$\cos x = -\frac{\sqrt{3}}{2}$$

$$\cos x = \frac{\sqrt{2}}{2}$$

$$\tan \frac{x}{2} = -1$$

$$\tan x = -1$$

$$\tan x = \sqrt{3}$$

$$\sqrt{2} \cos\left(2x - \frac{\pi}{4}\right) + 1 = 0$$

Equazioni trigonometriche riconducibili ad elementari

Equazioni risolubili mediante applicazione
della legge di annullamento del prodotto

$$\tan x(1 - \sin x) = 0$$

$$2 \sin^2 x - 3 \sin x = 0$$

$$\sin x = \sin 2x$$

Equazioni trigonometriche riconducibili ad elementari

Equazioni contenenti una sola
funzione goniometrica

$$2 \sin^2 x = 1$$

$$\sqrt{3} \tan^2 x - 4 \tan x + \sqrt{3} = 0$$

$$\frac{1}{1 + \cos x} - 2 = \frac{1}{\cos x - 1} + \frac{2}{3}$$

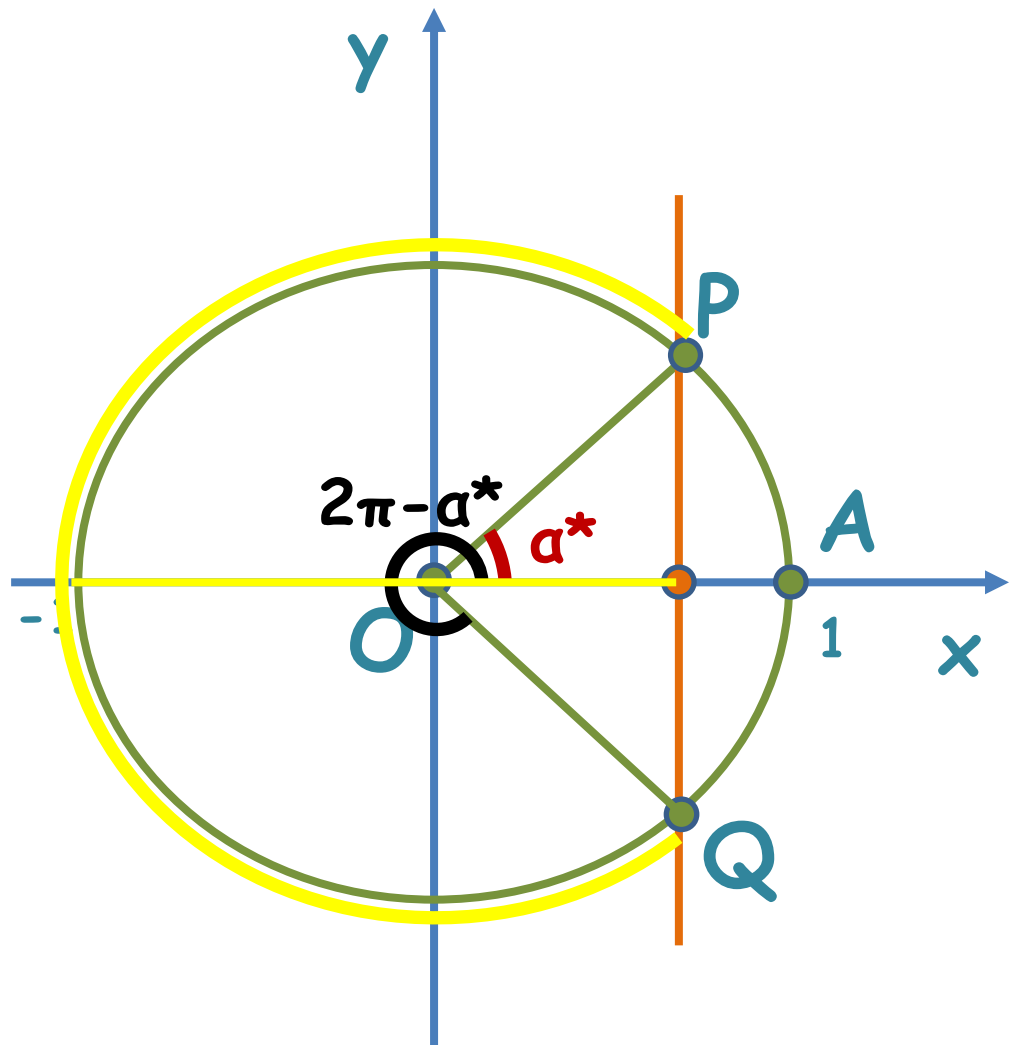
Equazioni trigonometriche riconducibili ad elementari

Equazioni riconducibili ad una
sola funzione goniometrica

$$2 \sin^2 x - 3 \cos x = 0$$

$$\tan x - \cot ax = \frac{2}{3} \sqrt{3}$$

Disequazioni trigonometriche elementari



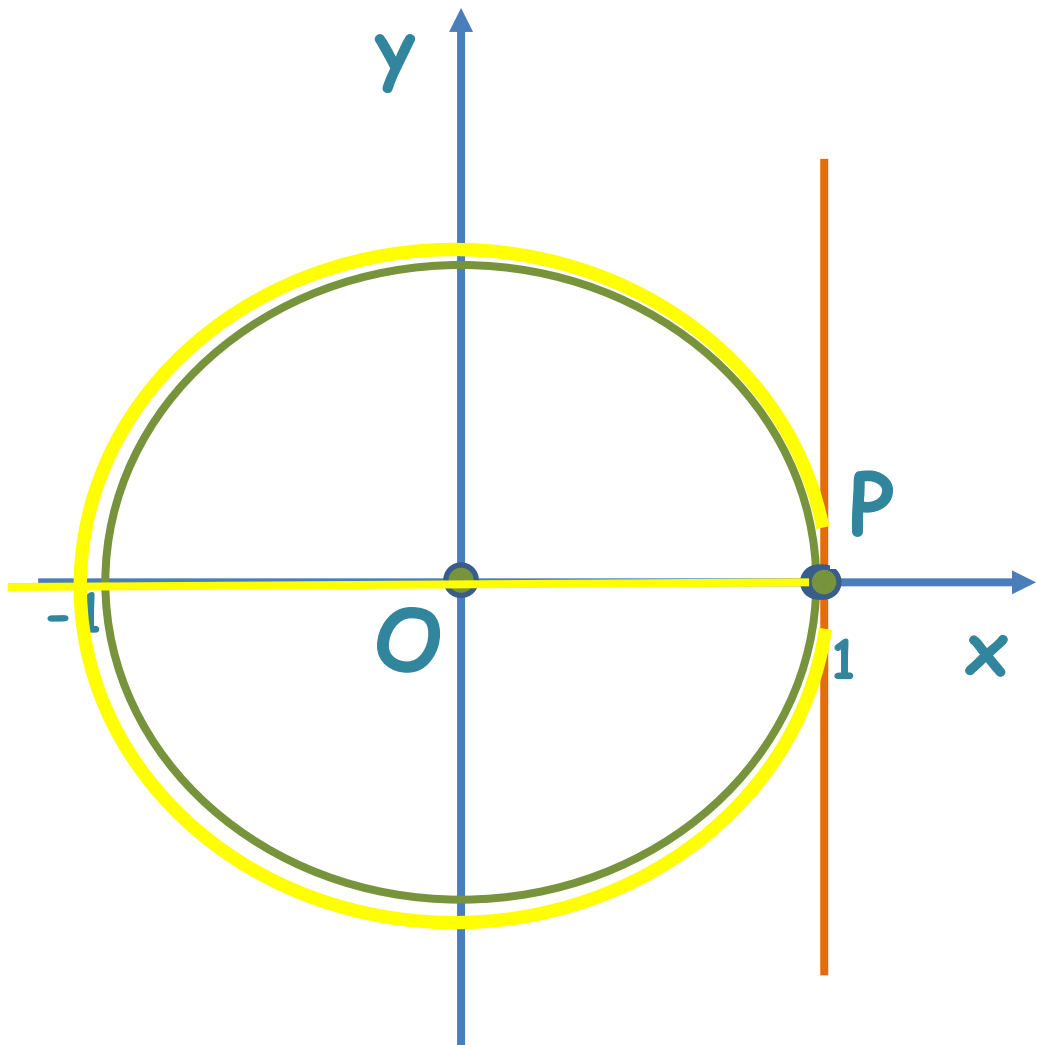
$$\cos \alpha < q$$

$$-1 < q < 1$$

$$\cos \alpha = q$$

Soluzione:
 $\alpha^* < \alpha < 2\pi - \alpha^*$

Disequazioni trigonometriche elementari

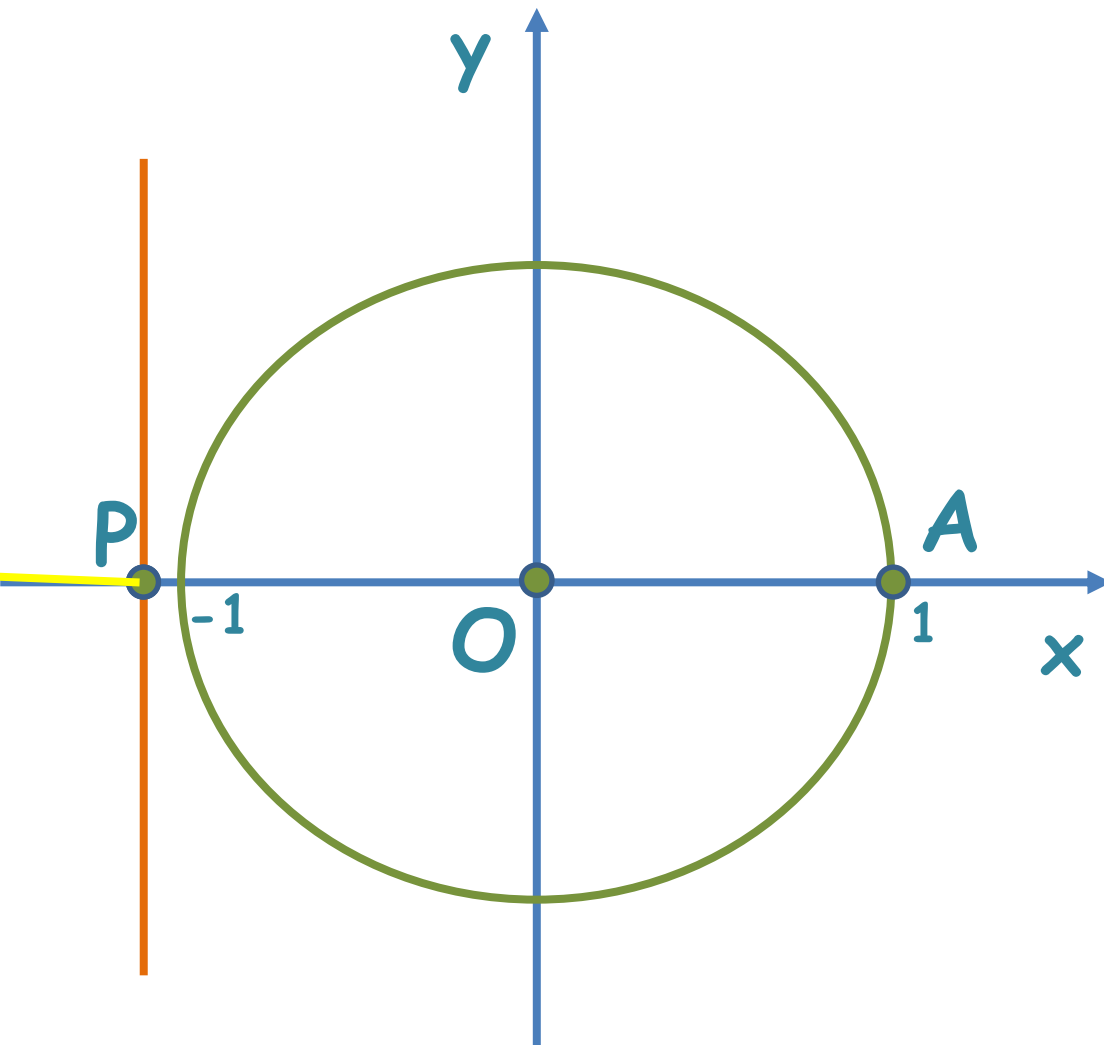


$$\cos \alpha < q$$

$-1 < q < 1$ **Soluzione:**
 $\alpha^* < \alpha < 2\pi - \alpha^*$

$q = 1$ **Soluzione:**
 $0 < \alpha < 2\pi$

Disequazioni trigonometriche elementari



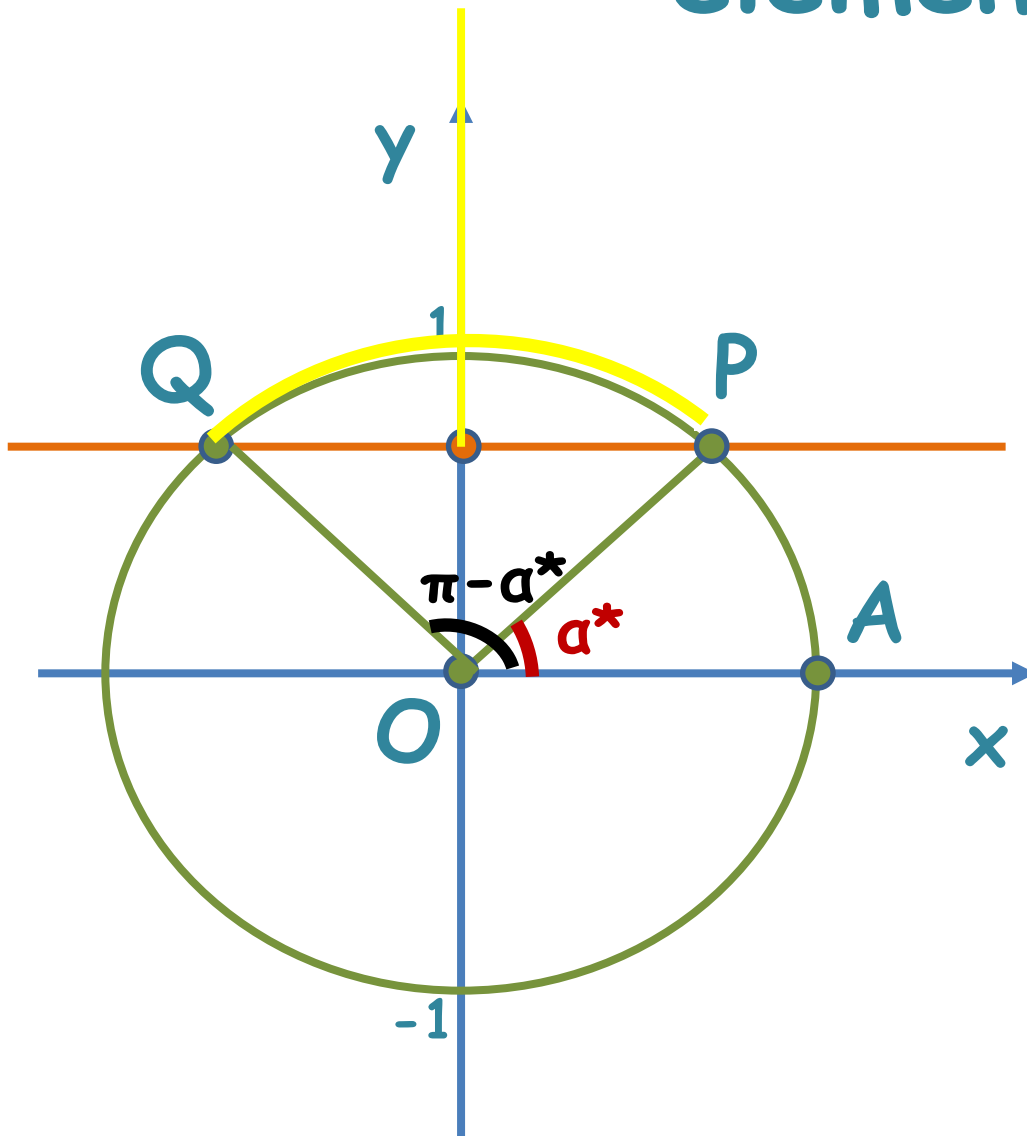
$$\cos \alpha < q$$

$$-1 < q < 1 \quad \text{Soluzione:} \\ \alpha^* < \alpha < 2\pi - \alpha^*$$

$$q = 1 \quad \text{Soluzione:} \\ 0 < \alpha < 2\pi$$

$$q \leq -1 \quad \text{Nessuna} \\ \text{soluzione}$$

Disequazioni trigonometriche elementari

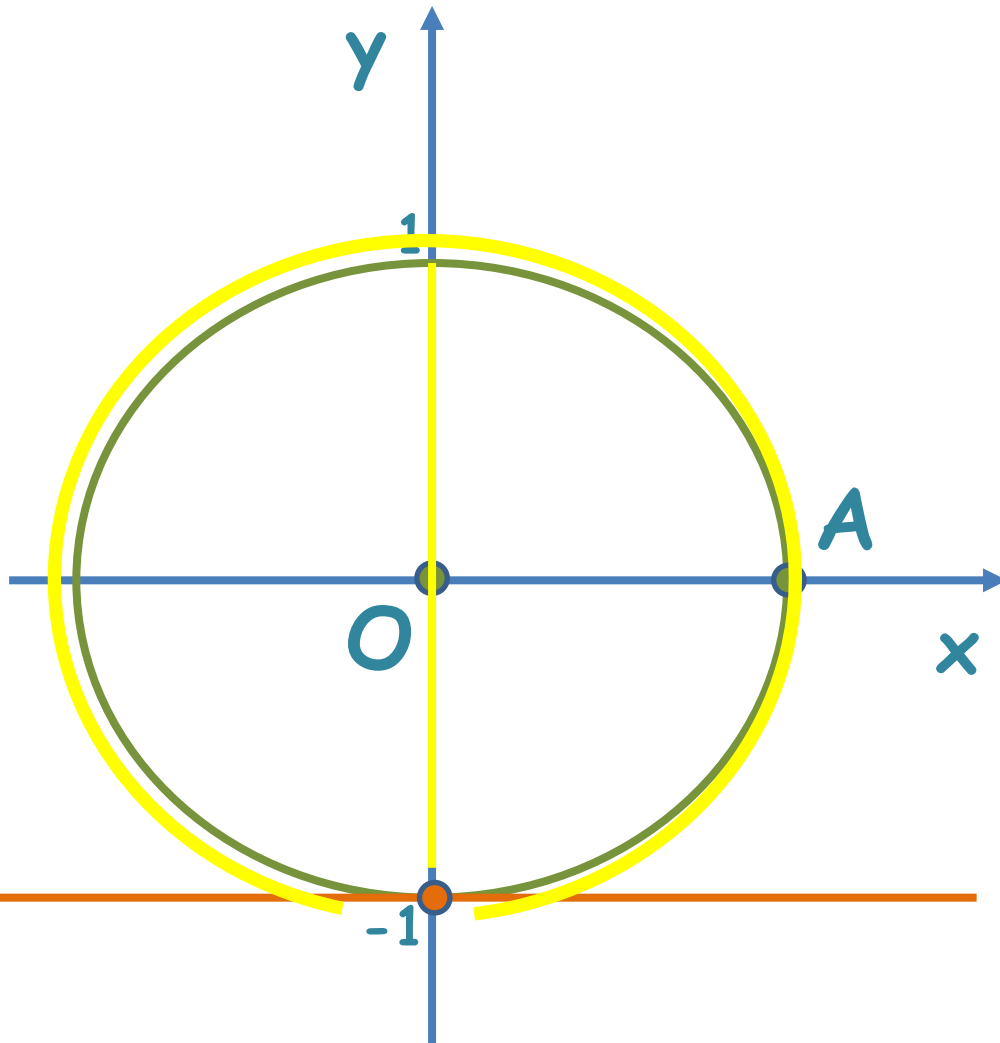


$$\sin \alpha > p$$

$$-1 \leq p < 1$$

1 soluzione:
 $\alpha^* < \alpha < \pi - \alpha^*$

Disequazioni trigonometriche elementari



$$\sin \alpha > p$$

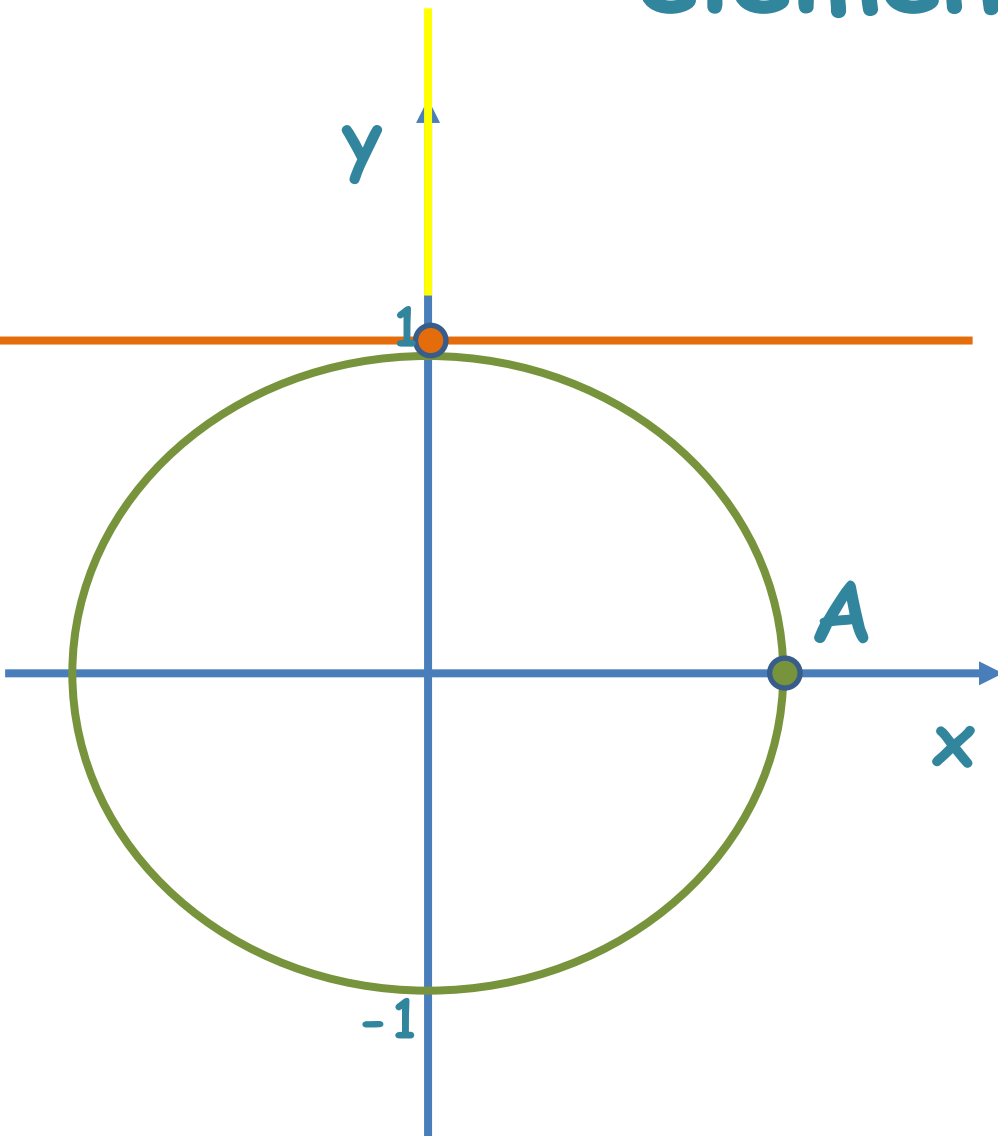
$$-1 \leq p < 1$$

1 soluzione:
 $\alpha^* < \alpha < \pi - \alpha^*$

$$p = -1$$

1 soluzione:
 $0 < \alpha < 3\pi/2 \cup$
 $3\pi/2 < \alpha < 2\pi$

Disequazioni trigonometriche elementari



$$\sin \alpha > p$$

$$-1 \leq p < 1$$

1 soluzione:
 $\alpha^* < \alpha < \pi - \alpha^*$

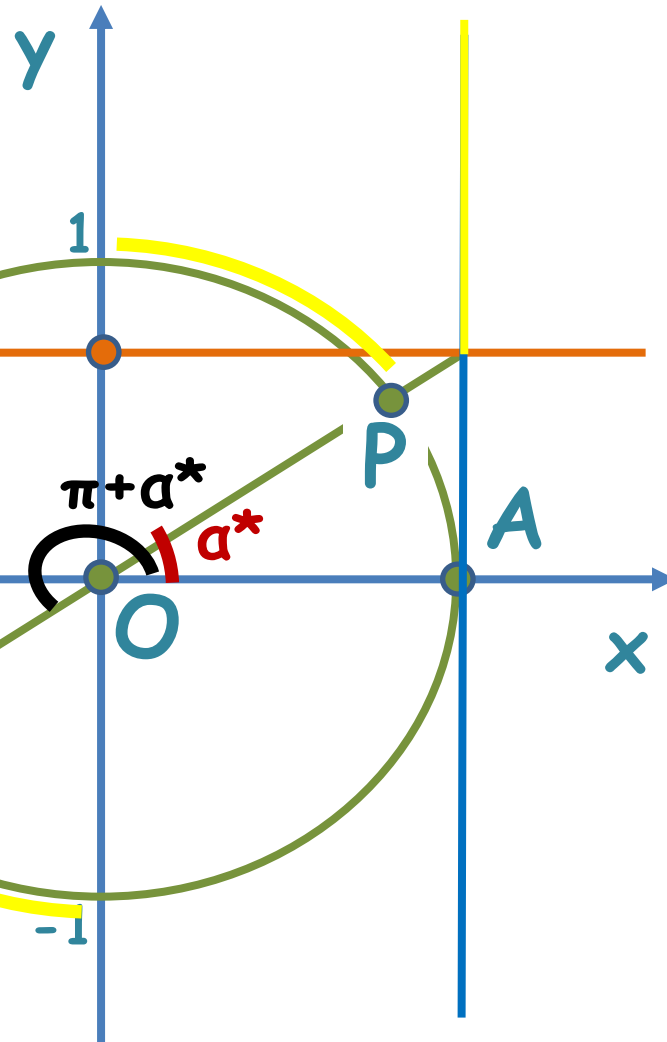
$$p = -1$$

1 soluzione:
 $0 < \alpha < 3\pi/2$ U
 $3\pi/2 < \alpha < 2\pi$

$$p = 1$$

Nessuna
soluzione

Disequazioni trigonometriche elementari



$$\operatorname{tg} \alpha > m$$

$$\forall m \in \mathbb{R}$$

Soluzione:

$$\alpha^* < \alpha < \pi/2 \cup$$

$$\pi + \alpha^* < \alpha < 3\pi/2$$

Disequazioni trigonometriche elementari

$$3 \tan x < \sqrt{3}$$

$$2 \sin x \geq 1$$

$$\frac{1}{2} < \cos x < \frac{\sqrt{2}}{2}$$