3

$$A = \begin{pmatrix} 4 & 6 & 0 \\ -3 & -5 & 0 \\ -3 & -6 & -5 \end{pmatrix}$$

$$\det A = -5 \begin{vmatrix} 4 & 6 \\ -3 & -5 \end{vmatrix} = -5(-20+18) = 10$$

$$\det A \neq 0 \Rightarrow r(A) = 3$$

$$A^{-1} = \frac{(\mathrm{cof}\,A)^T}{\det A} = \begin{pmatrix} 5/2 & 3 & 0 \\ -3/2 & -2 & 0 \\ 3/10 & 3/5 & -1/5 \end{pmatrix}$$

$$\mathrm{cof}\,A = \begin{pmatrix} 25 & -15 & 3 \\ 30 & -20 & 6 \\ 0 & 0 & -2 \end{pmatrix} \qquad (\mathrm{cof}\,A)^T = \begin{pmatrix} 25 & 30 & 0 \\ -15 & -20 & 0 \\ 3 & 6 & -2 \end{pmatrix}$$

Verifico che il risultato ottenuto sia corretto

$$A \cdot A^{-1} = \begin{pmatrix} 4 & 6 & 0 \\ -3 & -5 & 0 \\ -3 & -6 & -5 \end{pmatrix} \begin{pmatrix} 5/2 & 3 & 0 \\ -3/2 & -2 & 0 \\ 3/10 & 3/5 & -1/5 \end{pmatrix} =$$

$$= \begin{pmatrix} 10-9+0 & 12-12+0 & 0 \\ -15/2+15/2 & -9+10 & 0 \\ -15/2+9-15/10 & -9+12-3 & 5/5 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I$$

Gli autovalori di A sono quei numeri reali λ tali che

$$\det(A - \lambda I) = 0$$

$$A - \lambda I = \begin{pmatrix} 4-\lambda & 6 & 0 \\ -3 & -5-\lambda & 0 \\ -3 & -6 & -5-\lambda \end{pmatrix}$$

$$\det(A - \lambda I) = (-5-\lambda)\left[(4-\lambda)(-5-\lambda)+18\right]$$

$$= (-5-\lambda)(-20+5\lambda-4\lambda+\lambda^2+18)$$

$$= (-5-\lambda)(\lambda^2+\lambda-2) = (-5-\lambda)(\lambda+2)(\lambda-1)$$

$$\det(A-\lambda I) = 0 \iff \underbrace{\lambda = 1 \lor \lambda = -2 \lor \lambda = -5}_{\text{AUTOVALORI DELLA MATRICE } A}$$

PER OGNI AUTOVALORE DETERMINIAMO IL CORRISPONDENTE AUTOVETTORE

- $\lambda = 1$

$(A - \lambda I)\underline{v} = \underline{0}$

$$\begin{pmatrix} 3 & 6 & 0 \\ -3 & -6 & 0 \\ -3 & -6 & -6 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} 3v_1 + 6v_2 = 0 \\ \cancel{-3v_1 - 6v_2 = 0} \\ -3v_1 - 6v_2 - 6v_3 = 0 \end{cases} \qquad R_3 = R_3 + R_1 \qquad \begin{cases} \cancel{3}v_1 + \overset{2}{\cancel{6}}v_2 = 0 \\ \\ -6v_3 = 0 \end{cases}$$

$(-2t, t, 0)$

- $\lambda = -2$

$(A - \lambda I)\underline{v} = \underline{0}$

$$\begin{pmatrix} 6 & 6 & 0 \\ -3 & -3 & 0 \\ -3 & -6 & -3 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$(t, -t, t)$

$$\begin{cases} 6v_1 + 6v_2 = 0 \\ \cancel{-3v_1 - 3v_2 = 0} \\ -3v_1 - 6v_2 - 3v_3 = 0 \end{cases} \qquad R_3 = 2R_3 + R_1 \qquad \begin{cases} \cancel{6}v_1 + \cancel{6}v_2 = 0 \\ \\ -\cancel{6}v_2 - \cancel{6}v_3 = 0 \end{cases}$$

- $\lambda = -5$

$(A - \lambda I)\underline{v} = \underline{0}$

$$\begin{pmatrix} 9 & 6 & 0 \\ -3 & 0 & 0 \\ -3 & -6 & 0 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} 9v_1 + 6v_2 = 0 \\ -3v_1 = 0 \\ -3v_1 - 6v_2 = 0 \end{cases} \longrightarrow (0, 0, t)$$

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1) $\underline{u} = (4, -3, 1)$ $\quad \underline{v} = (-1, 1, -2)$ $\quad \underline{w} = (3, 0, 1)$

$$\underline{u} - \underline{v} + 2\underline{w} = (4, -3, 1) - (-1, 1, -2) + 2(3, 0, 1)$$

$$= (4, -3, 1) + (1, -1, 2) + (6, 0, 2) =$$

$$= (11, -4, 5)$$

$$\underline{u} \circ \underline{v} = 4(-1) + (-3) \cdot 1 + 1 \cdot (-2) = -4 - 3 - 2 = -9$$

$$\underline{v} \wedge \underline{w} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ -1 & 1 & -2 \\ 3 & 0 & 1 \end{vmatrix} = \bar{i} \begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} - \bar{j} \begin{vmatrix} -1 & -2 \\ 3 & 1 \end{vmatrix} + \bar{k} \begin{vmatrix} -1 & 1 \\ 3 & 0 \end{vmatrix}$$

$$= \bar{i}\,(1 - 0) - \bar{j}\,(-1 + 6) + \bar{k}\,(0 - 3)$$

$$= \bar{i} - 5\bar{j} - 3\bar{k}$$

$$= (1, -5, -3)$$

2 $z_1 = \sqrt{3} - 3i$ $\quad\quad z_2 = -4i$

$$z = (z_2)^2 + z_1\,(i - z_2) - \frac{24i}{z_1} + \left|\sqrt{-9}\right|$$

$$= (-4i)^2 + (\sqrt{3} - 3i)(i + 4i) - \frac{24i}{\sqrt{3} - 3i} + |3i|$$

$$= 16i^2 + (\sqrt{3} - 3i)(5i) - \frac{24i(\sqrt{3} + 3i)}{3 + 9} + 3$$

$$= -16 + 5\sqrt{3}i - 15i^2 - \frac{\overset{2}{\cancel{24}}}{\cancel{12}}(i\sqrt{3} + 3i^2) + 3$$

$$= -16 + 5\sqrt{3}i + 15 - 2\sqrt{3}i + 6 + 3$$

$$= 8 + 3\sqrt{3}i$$